# **The Finite Element Method**

## Div. of Solid Mechanics

# Course program, vt2, 2013

## **Course description**

The finite element method (FEM) is a numerical method able to solve arbitrary differential equations, i.e. boundary value problems. The method is today the most powerful numerical method within solid mechanics; this since arbitrary geometries and complex material models can be treated. Within the modern industry the finite element method is the key factor in many construction phases. Since the method is a solution method for any partial differential equations it can be used for any problem that is controlled by field equations, for instance heat conduction, diffusion, electromagnetism and solid mechanics.

The emphasis in the course in placed on the understanding of the fundamental principles of FEM and its numerical formulation. During the course the participants implement their own finite element program and thereby gain understanding of the method in detail.

## Lectures:

Monday 13-15, MH:B, (18/3,15/4,29/4,6/5,13/5,20/5) Tuesday 8-10, V:A, (9/4,7/5) Wednesday 8-10, V:A, (20/3,17/4,24/4,15/5,22/5) Thursday 8-10, E:A, (21/3,11/4,18/4)

Lectures: Mathias Wallin Div. of Solid Mechanics



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Problem sessions: Eric Borgqvist, Eric.Borgqvist@solid.lth.se



Jonas Engqvist, Jonas.Engqvist@solid.lth.se

Marcus Alexandersson



| FHI    | FHLF01, FHL064   |  |   |   |   |         |         |
|--------|--|--|---|---|---|---------|---------|
| v 12   | Må 18/3  | Ti 19/3  | On 20/3   | To 21/3   | Fr 22/3   | Lö 23/3 | Sö 24/3 |
| 8<br>9 |  | FHLF01, FHL084,<br>Ovn Ovn<br>M:Q MtM2<br>Finita Finita<br>elementmet<br>oden oden | FHL064, FHLF01, Förel<br>V:A<br>Finita elementmetoden<br>F3, M4-bem, M4-fo, M4- | FHL084, FHLF01, Förel<br>E:A<br>Finita elementmetoden<br>F3, M3, M4-bem, M4-fo,<br>M4-pu, E4, MD4, N4 | FHLF01, Övn<br>M-Q<br>Finita elementmetoden<br>F3.01-02                                 |         |         |
| 10     |  | F3.01-02 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,<br>0:00                              | 10.00   | 10:00   | 10:00-10:00   |         |         |
|        |  |  |   |   | HLUO4, OVN<br>M:M2<br>Finita elementmetoden<br>M4-bem, M4-fo, M4-pu,<br>E4, M3, MD4, N4 |         |         |
| 11     |  |  |   |   |   |         |         |
| 12     |  |  |   |   | 12:00   |         |         |
|        | 13:00<br>FHL064, FHLF01, Förel<br>MH:B<br>Finita elementmetoden<br>F3, M3, M4-bern, M4-fo, | 13.00-<br>FHLF01, FHL084,<br>Övn Övn<br>M:R M:M2<br>Finita Finita                  | ŀ   |   | 11 11 11 11 11 11 11 11 11 11 11 11 11  |         |         |
| 14     | M4-pu, E4, MD4, N4   | elementmet<br>oden<br>F3.03 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,                   |   |   | elementmet<br>oden oden<br>F3.03 M4-bem,<br>E4, M3,<br>MD4, N4                          |         |         |
| 15     | 15:00  | FHL064, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bem, M4-pu, E4, M3,<br>MD4, N4  | 1   |   | FHL064, Övn<br>MtM1<br>Finita elementmetoden<br>M4-bern, M4-pu, E4, M3,<br>MD4. N4      |         |         |
| 16     |  | 17:0   |   |   | 17:00   |         |         |

WEEK 2

#### FHLF01, FHL064

| v 15 | Må 8/4 | Ti 9/4   | On 10/4   | To 11/4                      | Fr 12/4   | Lö 13/4 | Sö 14/4 |
|------|--------|--|---|------------------------------|---|---------|---------|
| 8    |        | 08:00<br>FHL064, FHLF01, Förel<br>V:A<br>Finita elementmetoden<br>F3, M4-bern, M4-fo, M4-<br>pu, E4, M3, MD4, N4 | os:co<br>FHL064, Övn<br>M:M2<br>Finita elementmetoden<br>M4-bem, M4-fo, M4-pu,<br>E4, M3, MD4, N4 | E:A<br>Finita elementmetoden | 6500<br>FHLF01, Övn<br>M:Q<br>Finita elementmetoden<br>F3.01-02                           |         |         |
| 9    |        | 10:00  | 10:00   |                              | 10:00   |         |         |
| 10   |        |  |   |                              | FHL064, Övn<br>M:M2<br>Finita elementmetoden<br>M4-bern, M4-fo, M4-pu,<br>E4, M3, MD4, N4 |         |         |
| 11   |        |  |   |                              | 1200  |         |         |
| 12   |        | 13:00  |   |                              | 13:00   |         |         |
| 13   |        | FHL064, FHLF01,<br>Ovn Ovn<br>M:M2 M:R<br>Finita Finita  |   |                              | FHLF01, FHL064,<br>Ovn Ovn<br>M:R M:M1<br>Finita Finita                                   |         |         |
| 14   |        | elementmet<br>oden<br>M4-bem,<br>F3.03<br>M4-fo, M4-<br>pu, E4, M3,<br>15:00<br>15:00<br>15:00                   |   |                              | elementmet<br>oden<br>F3.03 M4-bem,<br>E4, M3,<br>MD4, N4                                 |         |         |
| 15   |        | FHL084, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bem, M4-pu, E4, M3,   |   |                              | FHL064, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bem, M4-pu, E4, M3,                    |         |         |
| 16   |        | MD4, N4  |   |                              | MD4, N4   |         |         |

| FHI  | LF01, FHL064   |   |   |   |   |         |         |
|------|--|---|---|---|---|---------|---------|
| v 16 | Må 15/4  | Ti 16/4   | On 17/4   | To 18/4   | Fr 19/4   | Lö 20/4 | Sö 21/4 |
| 8    |  | 0:00 0:00<br>FHLF01, FHL064,<br>Övn Övn<br>M:Q M:M2<br>Finita Finita<br>elementmet elementmet | peco<br>FHL064, FHLF01, Förel<br>V:A<br>Finita elementmetoden<br>F3, M4-bern, M4-fo, M4-<br>pu, E4, M3, MD4, N4 | os:oc<br>FHL064, FHLF01, Förel<br>E:A<br>Finita elementmetoden<br>F3, M3, M4-bern, M4-fo,<br>M4-pu, E4, MD4, N4 | os:oo<br>FHLF01, Övn<br>M:Q<br>Finita elementmetoden<br>F3.01-02                          |         |         |
| 9    |  | oden<br>F3.01-02 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,<br>10:00 10:00 10:00                    |   |   | 10:00   |         |         |
| 10   |  |   |   |   | FHL064, Övn<br>M:M2<br>Finita elementmetoden<br>M4-bern, M4-fo, M4-pu,<br>E4, M3, MD4, N4 |         |         |
| 11   |  |   |   |   | 1200  |         |         |
| 12   | .13:00   | 13:00   |   |   | 13:00   |         |         |
|      | MH:B<br>Finita elementmetoden<br>F3, M3, M4-bern, M4-fo, | FHLF01, FHL064,<br>Ovn Ovn<br>M:L1 M:M2<br>Finita Finita<br>elementmet elementmet             |   |   | FHLF01, FHL064,<br>Ovn Ovn<br>MtR M:M1<br>Finita Finita<br>elementmet elementmet          |         |         |
| 14   |  | oden<br>F3.03 0den<br>M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,<br>15:00 15:00 15:00               |   |   | oden<br>F3.03 M4-bern,<br>E4, M3,<br>MD4, N4  |         |         |
| 15   |  | FHL064, Ovn<br>M:M1<br>Finita elementmetoden<br>M4-bem, M4-pu, E4, M3,<br>MD4, N4             |   |   | FHL064, Övn<br>MtM1<br>Finita elementmetoden<br>M4-bern, M4-pu, E4, M3,<br>MD4, N4        |         |         |
| 16   |  | 17.00   |   |   | 17:00   |         |         |

WEEK 4

| v 17 | Må 22/4  | Ti 23/4  | On 24/4  | To 25/4 | Fr 26/4  | Lö 27/4 | Sö 28/4 |
|------|--|--|--|---------|--|---------|---------|
| 8    |  | M:Q M:M2<br>Finita Finita  | os:co-<br>FHL064, FHLF01, Förel<br>V:A<br>Finita elementmetoden<br>F3, M4-bern, M4-fo, M4-<br>ra, F4 M2 M24 M4 |         | 08:00<br>FHLF01, Övn<br>M:Q<br>Finita elementmetoden<br>F3.01-02                                       |         |         |
| 9    | 10:00  | elementmet<br>oden<br>F3.01-02 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,<br>-10:00 -10:00 |  |         | 1000   |         |         |
|      | FHL064, Övn<br>M:M2<br>Finita elementmetoden<br>M4-bern, M4-pu, E4, M3,<br>MD4, N4 |  | 1000   |         | FHL064, DatorÖvn<br>M:Ida, M:Ina4<br>Finita elementmetoden<br>M4-bem, M4-fo, M4-pu,<br>E4. M3, MD4, N4 |         |         |
| 11   | 12:00  |  |  |         | 200, MD, MD, W   |         |         |
| 12   |  |  |  |         |  |         |         |
| 13   |  | 1300-<br>FHLF01, FHL064,<br>DatorOvn Ovn<br>M:Em1-5 M:M2<br>Finita Finita            |  |         | 13:00-<br>FHLF01, FHL064,<br>DatorOvn Ovn<br>MtIda, MtM1<br>MtEm4-5 Finita                             |         |         |
| 14   |  | elementmet<br>oden<br>F3 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,                        |  |         | Finita elementmet<br>elementmet oden<br>M4-bem,<br>F3 E4, M3,<br>MD4, N4                               |         |         |
| 15   |  | FHL084, Övn<br>M:Em1-3<br>Finita elementmetoden<br>M4-bern, M4-fo, M4-pu,            |  |         | FHL064, Övn<br>MM1<br>Finita elementmetoden<br>M4-bern, M4-pu, E4, M3,                                 |         |         |
| 16   |  | E4, M3, MD4, N4  |  |         | MD4, N4  |         |         |

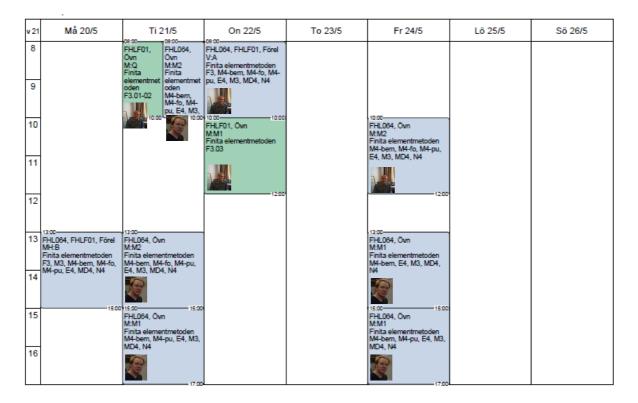
| v 18 | Må 29/4   | Ti 30/4 | On 1/5 | To 2/5 | Fr 3/5  | Lö 4/5 | Sö 5/5 |
|------|---|---------|--------|--------|---|--------|--------|
| 8    |   |         |        |        |   |        |        |
| 9    |   |         |        |        |   |        |        |
| 10   |   |         |        |        | FHL064, Övn<br>M:M2<br>Finita elementmetoden<br>M4-bem, M4-fo, M4-pu,           |        |        |
| 11   |   |         |        |        | E4, M3, MD4, N4   |        |        |
| 12   |   |         |        |        |   |        |        |
| 13   | 13:00-<br>FHL064, FHLF01, Förel<br>MH:B<br>Finita elementmetoden<br>F3, M3, M4-bern, M4-fo, |         |        |        | 13:00-<br>FHL064, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bern, E4, M3, MD4, |        |        |
| 14   |   |         |        |        | N4  |        |        |
| 15   | 15:00   |         |        |        | FHL064, Övn<br>MM1<br>Finita elementmetoden<br>M4-bern, M4-pu, E4, M3,          |        |        |
| 16   |   |         |        |        | MD4, N4   |        |        |

## WEEK 6

| v 19 | Må 6/5  | Ti 7/5   | On 8/5  | To 9/5 | Fr 10/5 | Lö 11/5 | Sö 12/5 |
|------|---|--|---|--------|---------|---------|---------|
| 8    |   | FHL064, FHLF01, Förel<br>V:A<br>Finita elementmetoden                                  | FHLF01, FHL064,<br>Ovn Ovn<br>MX1b M:M2<br>Finita Finita<br>elementmet elementmet |        |         |         |         |
| 9    |   |  | oden oden<br>F3.01-02 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,                        |        |         |         |         |
| 10   |   |  |   |        |         |         |         |
| 11   |   |  |   |        |         |         |         |
| 12   | .13:00  | 13:00-   |   |        |         |         |         |
|      | FHL064, FHLF01, Förel<br>MH:B<br>Finita elementmetoden<br>F3, M3, M4-bern, M4-fo. | FHLF01, FHL064,<br>Ovn Ovn Ovn<br>MiL1 M:M2<br>Finita Finita<br>elementmet elementmet. |   |        |         |         |         |
| 14   |   | oden<br>F3.03 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,<br>15:00 15:00                      |   |        |         |         |         |
| 15   |   | FHL084, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bem, M4-pu, E4, M3,<br>MD4, N4      |   |        |         |         |         |
| 16   |   | MD4, N4  |   |        |         |         |         |

| v 20 | Må 13/5  | Ti 14/5   | On 15/5  | To 16/5 | Fr 17/5   | Lö 18/5 | Sö 19/5 |
|------|--|---|--|---------|---|---------|---------|
| 8    |  | 08:00<br>FHLF01, FHL064,<br>Ovn Ovn<br>M:Q M:M2<br>Finita Finita                | FHL064, FHLF01, Förel<br>V:A<br>Finita elementmetoden<br>F3, M4-bern, M4-fo, M4- |         |   |         |         |
| 9    |  | elementmet<br>oden<br>F3.01-02<br>M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,          |  |         |   |         |         |
| 10   |  | 10:00   | 10:00  |         | 10:00<br>FHL064, Övn<br>M:M2<br>Finita elementmetoden<br>M4-bern, M4-fo, M4-pu, |         |         |
| 11   |  |   |  |         | E4, M3, MD4, N4   |         |         |
| 12   |  |   |  |         | 12:00   |         |         |
|      | MH:B<br>Finita elementmetoden<br>F3, M3, M4-bern, M4-fo, | 13:00-<br>FHLF01, FHL064,<br>Ovn Ovn<br>M:L1 M:M2<br>Finita Finita              |  |         | 13:00-<br>FHL064, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bern, E4, M3, MD4, |         |         |
| 14   |  | elementmet<br>oden<br>F3.03 M4-bem,<br>M4-fo, M4-<br>pu, E4, M3,<br>15:00 15:00 |  |         | N4  |         |         |
| 15   |  | FHL084, Övn<br>M:M1<br>Finita elementmetoden<br>M4-bem, M4-pu, E4, M3,          |  |         | FHL064, Övn<br>MtM1<br>Finita elementmetoden<br>M4-bern, M4-pu, E4, M3,         |         |         |
| 16   |  | MD4, N4   |  |         | MD4, N4   |         |         |

WEEK 8



EXAM WEEK Wed: 5/6: Vic: 1A-D, Vic 2A 14-19

## **Course literature**

Ottosen, Niels Saabye and Petersson, Hans: Introduction to the Finite Element Method, Prentice Hall.

#### Wallin, Mathias: "Introduction to the Finite

**Element Method- Exercises''**, Solid Mechanics, 2012. The exercises can be downloaded from the course website.

The course book can be bought at KFS.

Additional notes on transient problems is available at the course home-page.

#### **CALFEM-manual, computer program for learning the finite element method**, Structural mechanics and Solid Mechanics, Lund 1999.

The Matlab-toolbox CALFEM can be downloaded from our homepage (www.solid.lth.se).

## Assignment

The course includes a mandatory assignment. The assignment is performed in groups of two, or individually. **The assignment shall be handed in not later than May 27 at 16.00**. The assignment will be graded with up to 5 points which can be added to the points obtained at the exam June 5. Note that the bonus points is only valid at the exam June 5, 2013. A report that is handed in after May 27 will be given 0 points. The report must be approved not later than June 12.

### Submission

You should submit your report in PDF format to FHLF01@solid.lth.se or FHL064@solid.lth.se. In addition to your report you should also attach your m-files in the email. Moreover, a paper version should also be handed in to the division of Solid Mechanics.

### Examination

The examination of the course consists of a final examination and an assignment. The total points for passing the exam is 30. Total points is 60.

The exam takes place June 5, 13-19, Vic1A-D, VIC 2A

## Preliminary lecture schedule

| Lecture 1  | 18/3 | Introduction to FE-analysis, Chap. 1 and Chap. 2 |
|------------|------|--|
| Lecture 2  | 20/3 | Chap. 3  |
| Lecture 3  | 21/3 | Chap. 4  |
| Lecture 4  | 9/4  | Chap. 5 FHL064                                   |
| Lecture 5  | 11/4 | Chap. 6, Chap 7 (1D)                             |
| Lecture 6  | 15/4 | Chap. 7 (3D), Chap 8                             |
| Lecture 7  | 17/4 | Chap. 9  |
| Lecture 8  | 18/4 | Chap. 10 + Transient heat flow, Chap 11          |
| Lecture 9  | 24/4 | Chap. 12, Chap. 13                               |
| Lecture 10 | 29/4 | Chap. 15 and 16                                  |
| Lecture 11 | 6/5  | Chap. 19   |
| Lecture 12 | 7/5  | Chap. 20   |
| Lecture 13 | 13/5 | Variational principles. FHLF01                   |
| Lecture 14 | 15/5 | Chap. 17. FHL064                                 |
| Lecture 15 | 20/5 | Guest lecture. Jan Granlund                      |
| Lecture 16 | 22/5 | Reserv   |

## Exercises

| Exercise 1  | Chap. 2                            |
|-------------|------------------------------------|
| Exercise 2  | Chap. 3                            |
| Exercise 3  | Chap. 4.                           |
| Exercise 4  | Chap. 5, Chap. 6                   |
| Exercise 5  | Chap. 7, Chap. 8                   |
| Exercise 6  | Chap. 9 (not 9.4 and 9.5)          |
| Exercise 7  | Chap. 10                           |
| Exercise 8  | 9.4, 9.5 and Chap. 11              |
| Exercise 9  | Chap. 12, Chap. 13                 |
| Exercise 10 | Chap. 15, Chap. 16                 |
| Exercise 11 | Chap. 19                           |
| Exercise 12 | Chap. 20                           |
| Exercise 13 | Chap. 17 / Variational principles. |
| Exercise 14 | Consultation                       |
|             |                                    |