Assignment in
The Finite Element Method, 2014

Division of Solid Mechanics

The task is to write a finite element program and then analyse the temperature and stress distribution of a two dimensional structure. The problem should be solved using Matlab along with suitable subroutines included in the CALFEM toolbox. A well structured report that presents the findings should be written.

Main task

The cross section of a gas turbine stator blade should be dimensioned to sustain a given external load. The task is to analyse the temperature and stress distribution of such a structure during testing of the blade. A sketch of a simplified turbine cross section with two cooling channels can be seen in Figure 1.

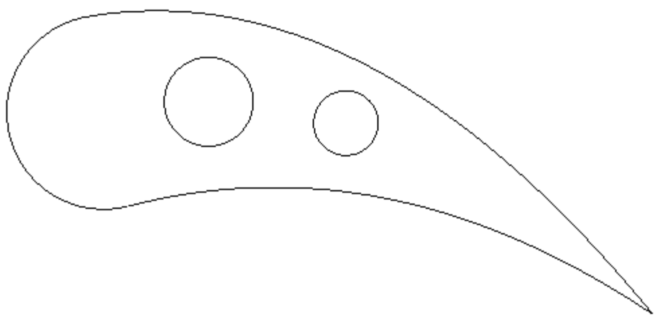

Figure 1: Simplified geometry

In the situation to be analysed the blade is assumed to be fixed at two discrete points and internally cooled by water in two internal channels. Due to the surrounding airflow an external pressure $p = p(x, y)$ is present along the boundaries, given by

$$p(x, y) = p_0 \left(1 - \frac{y}{a} - \frac{x}{b}\right) \tag{1}$$

The initial temperature of the blade is T_0 and the blade is then heated by convection between the blade and the surrounding airflow. The temperature of the ambient gas is T_∞ and the convection coefficient is α_c. The blade is assumed to consist of an intermetallic TiAl-based alloy specially designed to withstand the high temperatures present in a turbine. The material data needed to solve the assignment (such as Youngs modulus E, Poison's ratio ν, specific heat c etc.) is given in [1] and in Table 1.

To simplify the analysis the blade may be modelled using plane strain conditions and assumed to be isotropic. In Fig. 2 the dimensions of the geometry and boundary conditions as well as thermal and mechanical loading can be seen (the pressure p acts normal to the surface on all outer boundaries).

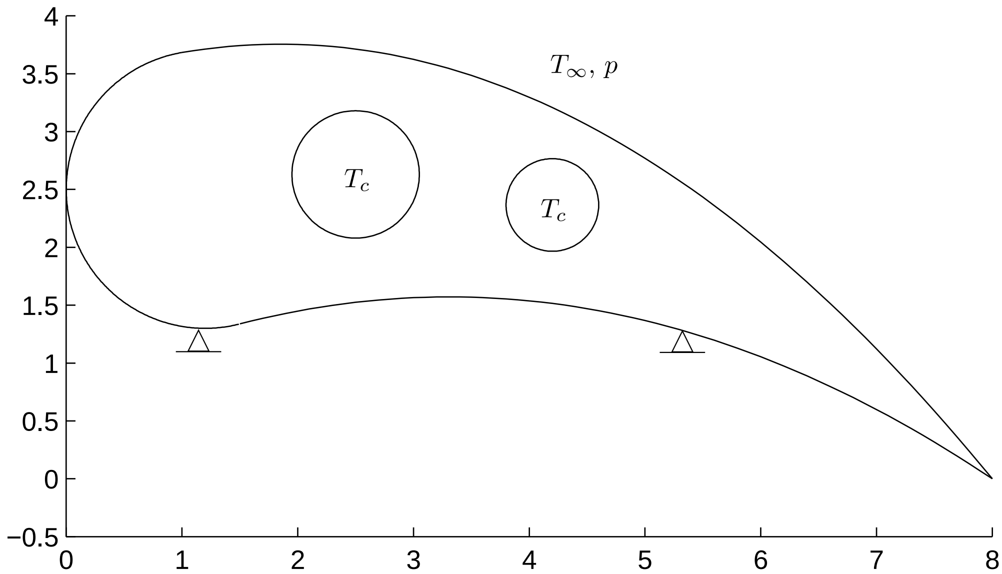

Figure 2: Geometry and boundary conditions. Dimensions in cm.

$\nu = 0.3$	$\rho = 3800$ kg/m^3	$\alpha_c = 50$ W/m^{2}K
$T_\infty = 700°$C	$T_0 = 20°$C	$T_c = 90°$C
$p_0 = 0.1$ MPa	$a = 0.1$ m	$b = 0.1$ m

Table 1: Numerical data

Problem formulation

i) Determine how the temperature distribution changes during operation. The temperature field should be presented at at least two representative time steps.

ii) Determine the effective stress (von Mises) field at equilibrium. The stress distribution should be presented for the pressure load, the thermal load and combined thermal and pressure load.

Procedure

A fully implicit time integration scheme should be used. Note that the element function for forming, $\boldsymbol{C}^e$, is available at the course homepage. A suitable element is the linear triangular element. At least 80 elements should be used when making the mesh. As a starting point, you have the strong formulation of the heat and the structural problems. A well structured report briefly containing all steps from the strong formulation to the FE formulation is to be returned to the Div. of Solid Mechanics no later than *2014 May 26 at 16.00*. The reader of the report is assumed to have the same knowledge level as the author. If the report contains theoretical errors, the report is returned in order to be corrected. It is possible to obtain up to 5 points which are augmented to the final points. The assignment should in any case be approved no later than *2014 June 12*. You should submit your report in PDF format to FHLF01@solid.lth.se or FHL064@solid.lth.se. In addition to your report you should also attach your m-files in the email. Moreover, a paper version should also be handed in to the division of Solid Mechanics. Note that the bonus points obtained is only valid for the examination in May 2014.

Hints:

1. A file containing the the analytical description of the profile is available on the course homepage. In order to facilitate the meshing you are allowed to simplify the geometry slightly so that the curved boundaries are replaced by piecewise linear boundaries.

2. The nodal extrapolation of the von Mises effective stress can be done according to:

```
for i=1:size(coord,1)
        [c0,c1]=find(Edof(:,2:4)==i);
        Seff_nod(i,1)=sum(Seff_el(c0))/size(c0,1);
end
```

where *Seff_nod* and *Seff_el* is the von Mises effective stress at the nodal points and in the elements, respectively. Note that here the topology matrix (Edof) is associated with the scalar problem. The von Mises stress is defined as:

$$\sigma_{eff} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{xx}\sigma_{zz} - \sigma_{yy}\sigma_{zz} + 3\tau_{xy}^2 + 3\tau_{xz}^2 + 3\tau_{yz}^2}$$

Collaboration

The task should be solved in groups of *two* or *individually*. For further details regarding collaboration see www.solid.lth.se and navigate to the course homepage.

Report

A fundamental ingredient in all research is that it should be possible to regenerate the results obtained based on the report. In the present situation this implies that the appended Matlab code should only be considered as supporting material, i.e. it should be possible to regenerate your results on the basis on the information provided in the report. Moreover, note that one variable for grading the report is the structure of the computer code, i.e. you should choose suitable names for variables etc. A suitable structure for the report is:

- **Introduction**: Description of the problem, geometry etc. Keep this section as short as possible.
- **Procedure**: Describe the solution procedure. Present the derivation of the weak form. Discuss how boundary conditions, initial conditions, thermal strains etc. are implemented. Note that you are encouraged to make references to textbooks but it is important to carefully present all calculations not available in the literature.
- **Results**: Present the results in illustrative figures and/or tables. Note that the results should be commented such that the reader can not misunderstand the results (correct labels, units, captions etc.)
- **Discussion**: This section should include a brief discussion of the results. Moreover, it is also suitable to add a discussion of for example error sources and possible improvements of the model.
- **References**: A reference list is mandatory.
- **Computer code**: Note that the code should be submitted along with your report. It is of major importance that variable names and function names are given descriptive names. Also the code should be well structured and clearly commented.

References

[1] W. J. Zhang, B. V. Reddy and S. C. Deevi. Physical properties of TiAl-base alloys, *Scripta Materialia*, 45:645-51, 2001.