Assignment in
The Finite Element Method, 2011

Division of Solid Mechanics

The task is to write a finite element program and then analyse the temperature and stress distribution of a structure. The problem should be solved using Matlab along with suitable subroutines included in the CALFEM toolbox. A well structured report that presents the findings should be written.

Main task

The walls of a containment building surrounding a nuclear reactor are dimensioned in order to prevent radioactive leakage and to resist a certain amount of inner pressure before breaking. The task is to analyse the temperature distribution and stress distribution of such a structure in the event of a meltdown. In the situation to be analysed there is a constant heat flow $q_n = 1200$ J/m^2 s from the reactor. Due to leakage and generated heat, an internal pressure P_{in} given by

$$P_{in} = \frac{P_0}{T_0}(T_m - T_0) + P_0 \tag{1}$$

is present in the air surrounding the reactor. The parameters P_0 and T_0 represent initial pressure and temperature (units Pa and $°C$), respectively. The mean temperature T_m inside the building is given by

$$T_m = \frac{1}{V}\int_V T dV \tag{2}$$

where the volume V is the space occupied by the air. The containment building is pre-stressed and this is modelled by a constant pressure P_{out} on the outside of the building. The structure is assumed to be rigidly attached to the ground. Convection occurs between the containment and the surrounding air and the convection coeffcient is α_c. The surrounding air is assumed to have the temperature T_∞. The density of the air is given by ρ_{air}. The building is made out of concrete and is assumed to be isotropic with Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α, density ρ_{con}, specific heat c_{con} and thermal conductivity k_{con}. The thermal conductivity of air, k_{air} has been determined to $k_{air} = 1.3$ W/(mK). Note that the air inside the containment building is assumed to not circulate.

Both the entire building (air and concrete) has an initial temperature of T_0 before the meltdown.

To simplify the analysis the containment building may be modeled using plane stress conditions. The structure is symmetric and to reduce the computational cost, use should be made out of this fact. In Fig. 1 the dimensions of the geometry, thermal and mechanical loading are shown.

Problem formulation

a) Determine how the temperature distribution changes during the meltdown. The temperature field should be presented after 10 minutes and after 4 hours.

b) Determine the effective stress (von Mises) field after 4 hours. The stress distribution should be presented for the pressure load, the thermal load and combinded thermal and pressure load.

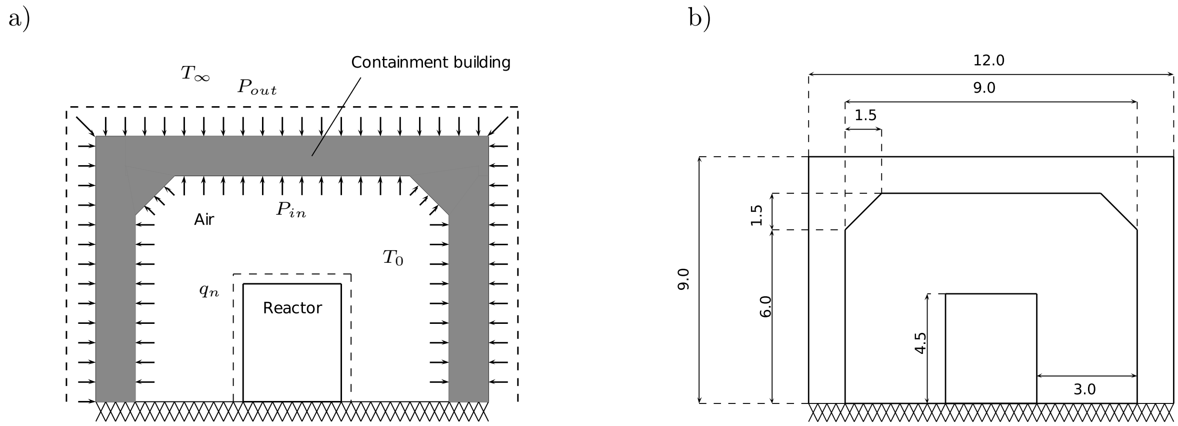

Figure 1: Sketches of the containment building, the dimensions in b) is in m.

Procedure

A fully implicit time integration scheme should be used. Note that the element function for forming, $\boldsymbol{C}^e$, is available at the course homepage. A suitable element is the linear triangular element. At least 80 elements should be used when making the mesh. As a starting point, you have the strong formulation of the heat and the structural problems. A well structured report briefly containing all steps from the strong formulation to the FE formulation is to be returned to the Div. of Solid Mechanics no later than **2011 May 20 at 16.00**. The reader of the report is assumed to have the same knowledge level as the author. If the report contains theoretical errors, the report is returned in order to be corrected. It is possible to obtain up to 5 points which are augmented to the final points. The assignment should in any case be approved no later than 2011-06-10. You should submit your report in PDF format to FHLF01@solid.lth.se or FHL064@solid.lth.se. In addition to your report you should also attach your m-files in the email. Moreover, a paper version should also be handed in to the division of Solid Mechanics. Note that the bonus points obtained is only valid for the examination in June 2011.

Hints:
1. Both the air and the building should be meshed in the thermal problem whereas only the concrete building should be included in the structural analysis. When creating the finite element mesh, number the elements and nodes of the concrete building first and then the air inside the building. This will make it easier to separate the structural and thermal problem.

2.The nodal extrapolation of the von Mises effective stress can be done according to:

```
for i=1:size(coord,1)
        [c0,c1]=find(Edof(:,2:4)==i);
        Seff_nod(i,1)=sum(Seff_el(c0))/size(c0,1);
end
```

where *Seff_nod* and *Seff_el* is the von Mises effective stress at the nodal points and in the elements, respectively. Note that here the topology matrix (Edof) is associated with the scalar problem. Note that the von Mises stress is defined as:

$$\sigma_{eff} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{xx}\sigma_{zz} - \sigma_{yy}\sigma_{zz} + 3\tau_{xy}^2 + 3\tau_{xz}^2 + 3\tau_{yz}^2}$$

Collaboration

The task should be solved in groups of *two* or *individually*. For further details regarding collaboration see www.solid.lth.se and navigate to the course homepage.

Report

A fundamental ingredient in all research is that it should be possible to regenerate the results obtained based on the report. In the present situation this implies that the appended matlab code should only be considered as supporting material, i.e. it should be possible to regenerate your results on the basis on the information provided in the report. Moreover, note that one variable for grading the report is the structure of the computer code, i.e. you should choose suitable names for variables etc. A suitable structure for the report is:

- **Introduction**: Description of the problem, geometry etc. Keep this section as short as possible.
- **Procedure**: Describe the solution procedure. Describe how the weak form is derrived. Discuss how boundary conditions, initial conditions, thermal strains etc. are implemented. Note that you are encouraged to make references to textbooks etc. It is important to carefully present all calculations that are not available in the litterature.
- **Results**: Present the results in illustrative figures and/or tables. Note that the results should be commented such that the reader can not missunderstand the results (correct labels, units, captions etc.)
- **Disscusion**: This section should include a brief disscusion of the results. Moreover, it is also suitable to add a discussion of for example error sources, possible improvements in the model.
- **Computer code**: Note that the code should be submitted along with your report. It is of major importance that variable names and function names are given descriptive names. The code should be well structured and clearly commented.

Numerical data for main task

$E = 30\ GPa$	ν=0.3	α=1 $\cdot 10^{-5}\ 1/K$
k_{con} = 10 W/(mK)	k_{air} = 1.3 W/(mK)	α_c=50 W/m^2K
T_∞= 5 ^oC	$P_0 = 0.1\ MPa$	P_{out}=0.5 MPa
T_0 = 20 ^oC	c_{con} = 2000 J/(KgK)	c_{air} = 1200 J/(KgK)
ρ_{con}=2400 Kg/m^3	ρ_{air} = 0.4 Kg/m^3	q_n = 1200 J/sm^2

Table 1: Numerical data