Assignment in
The Finite Element Method, 2012

Division of Solid Mechanics

The task is to write a finite element program and then analyse the temperature and stress distribution in a structure. The problem should be solved with Matlab along with suitable subroutines included in the CALFEM toolbox. A well structured report that presents the findings should be written.

Main task

The braking effect in a disc brake is obtained from brake pads that are pressed against a rotating disc. The task is to analyze the temperature and stress distribution in such a brake pad. Sketches of the brake pad are shown in Fig. 1. The pressure P is applied on top of the pad as shown in the figure, the surface that the pressure acts on have constant temperature T_g. The pad is pressed towards a rotating disc, the pressure gives rise to a friction-force, F_f, between the pad and the disc. The friction force is assumed to be uniformly distributed and fully developed. Since sliding occurs the friction force results in a heat generation Q, which is assumed to be uniformly distributed. The disc is assumed to be rigid, which implies that the contact between the pad and the disc can be modeled by applying displacement boundary conditions. The horizontal movement of the pad is prevented by the structure in the middle of the pad (the hatched part in Fig. 1).

The surrounding air is assumed to have the temperature T_∞ and the heat transport between the brake pad and the surrounding air is due to convection (convection coefficient α_c). The convection takes place on the boundaries marked with dashed lines in Fig. 1. The pad is made out of asbestos-free friction material BK4200 (asf) and is assumed to be isotropic with Young's modulus E_{asf}, Poisson's ratio ν_{asf}, thermal expansion coefficient α_{asf}, density ρ_{asf}, specific heat c_{asf} and thermal conductivity k_{asf}. The steel is isotropic with Young's modulus E_{steel}, Poisson's ratio ν_{steel}, thermal expansion coefficient α_{steel}, density ρ_{steel}, specific heat c_{steel} and thermal conductivity k_{steel}.

Both the brake pad and the steel structure is assumed to have the initial temperature T_∞, see table 1.

To simplify the analysis the brake pad may be modeled using plane stress conditions. In Fig. 2 the dimensions of the geometry, thermal and mechanical loading are shown. The thickness of the pad is 30 mm. A suitably time step is 0.5 s.

Problem formulation

a) Determine how the temperature distribution changes during the braking process. The temperature field should be presented after 30 seconds and 1 minute using contour plots.

b) Determine the effective stress (von Mises) field after 1 minute. The stress distribution should be presented for thermal load, mechanical load and combined thermal and mechanical load.

Procedure

A fully implicit time integration scheme should be used. Note that the element function for forming, $\boldsymbol{C}^e$, is available at the course homepage. A suitable element is the linear triangular

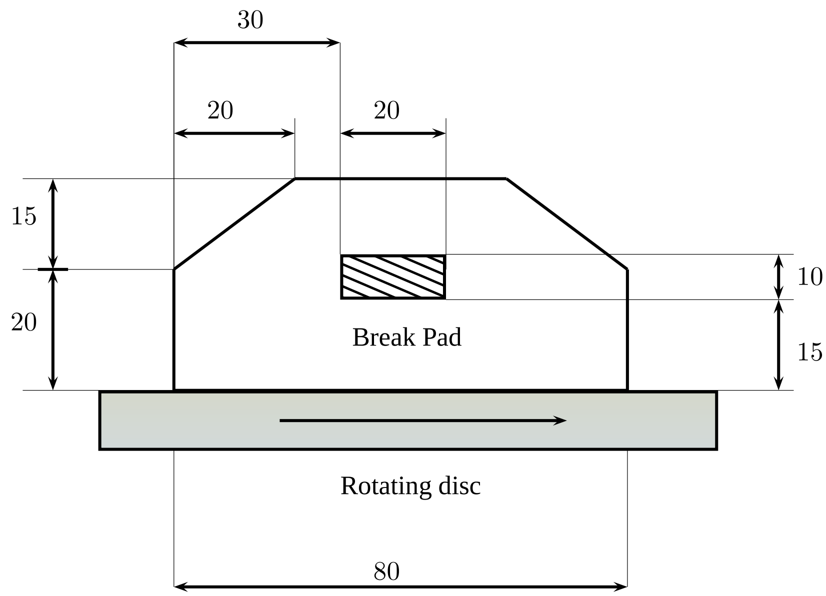

Figure 1: Sketch of the brake pad , the dimensions is in mm. The thickness of the pad is 30 mm.

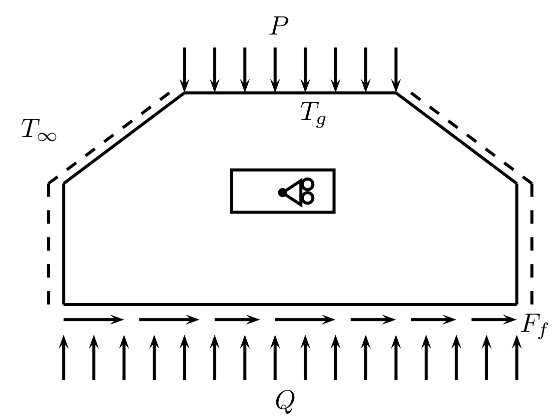

Figure 2: Sketch of the brake pad.

element. As a starting point, you have the strong formulation of the heat and the mechanical problems. A well structured report briefly containing all steps from the strong formulation to the FE formulation is to be returned to the Div. of Solid Mechanics no later than **2012 May 21 at 16.00**. The reader of the report is assumed to have the same knowledge level as the author. If the report contains theoretical errors, the report is returned in order to be corrected. It is possible to obtain up to 5 points which are augmented to the points obtained at the exam May 2012. The assignment should be approved no later than 2012-06-10. You should submit your report in PDF format to FHLF01@solid.lth.se or FHL064@solid.lth.se. In addition to your report you should also attach your m-files in the email. Moreover, a paper version should also be handed in to the division of Solid Mechanics. Note that the 5 bonus points obtained is only valid for the examination in May 2012.

Hints:
The nodal extrapolation of the von Mises effective stress can be done according to:

```
for i=1:size(coord,1)
```

```
        [c0,c1]=find(Edof(:,2:4)==i);
        Seff_nod(i,1)=sum(Seff_el(c0))/size(c0,1);
end
```

where *Seff_nod* and *Seff_el* is the von Mises effective stress at the nodal points and in the elements, respectively. Note that here the topology matrix (Edof) is associated with the scalar problem. Note that the von Mises stress is defined as:

$$\sigma_{eff} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{xx}\sigma_{zz} - \sigma_{yy}\sigma_{zz} + 3\tau_{xy}^2 + 3\tau_{xz}^2 + 3\tau_{yz}^2}$$

Collaboration

The task should be solved in groups of *two* or *individually*. For further details see www.solid.lth.se and navigate to the course page.

Report

A fundamental ingredient in all research is that it should be possible to regenerate the results obtained based on the report. In the present situation this implies that the appended matlab code should only be considered as supporting material. Moreover, note that one variable for grading the report is the structure of the computer code, i.e. you should choose suitable names for variables etc. A suitable structure for the report is:

- **Introduction**: Description of the problem, geometry etc. Keep this section as short as possible.
- **Procedure**: How the problems are solved (weak formulation, application of boundary conditions, thermal strains etc.). Note that you are encouraged to make references to text-books etc. It is important to carefully present all calculations that are not available in the litterature.
- **Results**: Present the results in illustrative figures and/or tables. Note that the results should be commented such that the reader can not missunderstand the results (correct labels, units, figure texts etc.)
- **Disscusion**: A brief disscusion about the results
- **Computer Code**

Numerical data for main task

$E_{asf} = 95\ GPa$	$\nu_{asf} = 0.29$	$\alpha_{asf} = 1 \cdot 10^{-4}\ 1/K$
$E_{steel} = 210\ GPa$	$\nu_{steel} = 0.3$	$\alpha_{steel} = 1 \cdot 10^{-5}\ 1/K$
$k_{asf} = 30$ W/(mK)	$k_{steel} = 45$ W/(mK)	$\alpha_c = 50\ W/m^2K$
$T_\infty = 25\ ^oC$	$T_g = 25\ ^oC$	$P = 50\ MPa$
$F_f = 50\ kN$	$c_{asf} = 1200$ J/(KgK)	$c_{steel} = 1000$ J/(KgK)
$\rho_{asf} = 2400\ Kg/m^3$	$\rho_{steel} = 6000\ Kg/m^3$	$Q = 7 \cdot 10^5$ J/m^2

Table 1: Numerical data