Assignment in
The Finte Element Method, 2013

Division of Solid Mechanics

The task is to develop a finite element program in order to analyze a complex geometry exposed to thermal and mechanical loads. This problem should be solved using Matlab and suitable subroutines included in CALFEM.

Main task

An engine containing a single cylinder is, inside the combustion chamber, exposed to an internal pressure p and a temperature T. A water cooling system consisting of four water channels (all with the depth d) is used to keep the temperature down. The cooling water is assumed to have constant temperature T_{water}. The surrounding air is assumed to have the temperature T_{∞} and the heat transport between the engine and the surrounding air is due to convection (convection coefficient α_c). The material is assumed to be isotropic with Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α and thermal conductivity k.

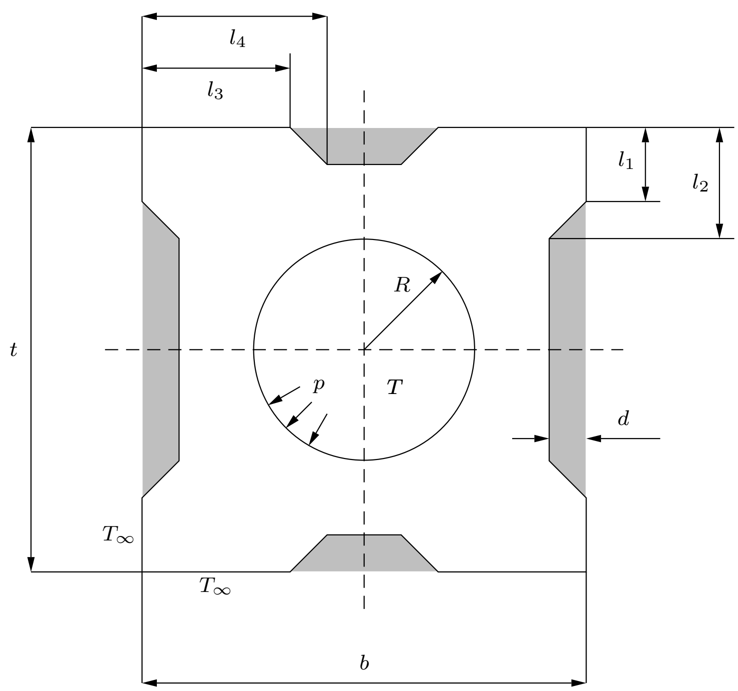

Figure 1: Cross section of engine

To be able to perform the analysis in CALFEM, the engine may be modeled using plane stress conditions. In Fig. 1 the dimensions of the geometry, thermal and mechanical loading are shown.

The cooling water is indicated by gray color.

Problem formulation

Determine the transient temperature evolution when the engine is started. Assume that the initial temperature is T_{ini} which for simplicity is taken the same as T_{water}. Contour plots showing the temperature evolution should be presented.

Determine the stationary temperature distribution and the corresponding stress distribution of the engine. The results should be presented using contour plots based on suitable extrapolations to nodal points. The stress analysis should be presented for thermal load, mechanical load and combined thermal and mechanical load.

Procedure

A fully implicit time integration scheme should be used. Note that the element function for forming, $\boldsymbol{C}^e$, is available at the course homepage. A suitable element is the linear triangular element. As a starting point, you have the strong formulation of the heat and the mechanical problems. A well structured report briefly containing all steps from the strong formulation to the FE formulation is to be returned to the Div. of Solid Mechanics no later than **2013 May 27 at 16.00**. The reader of the report is assumed to have the same knowledge level as the author. If the report contains theoretical errors, the report is returned in order to be corrected. It is possible to obtain up to 5 points which are augmented to the points obtained at the exam June 2013. The assignment should be approved no later than 2013-06-12. You should submit your report in PDF format to FHLF01@solid.lth.se or FHL064@solid.lth.se. In addition to your report you should also attach your m-files in the email. Moreover, a paper version should also be handed in to the division of Solid Mechanics. Note that the bonus points obtained is only valid for the examination in June 2013.

The contour plots of the stress distribution is based on the stress at the nodal points. The extrapolation of the stress in the elements to the nodal points can be done by taking the mean value of the stresses in the elements connected to a node. The nodal extrapolation of the von Mises effective stress can be done according to:

```
for i=1:size(coord,1)
        [c0,c1]=find(Edof(:,2:4)==i);
        Seff_nod(i,1)=sum(Seff_el(c0))/size(c0,1);
end
```

where *Seff_nod* and *Seff_el* is the von Mises effective stress at the nodal points and in the elements, respectively. Note that here the topology matrix (Edof) is associated with the scalar problem. Note that the von Mises stress is defined as:

$$\sigma_{eff} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{xx}\sigma_{zz} - \sigma_{yy}\sigma_{zz} + 3\tau_{xy}^2 + 3\tau_{xz}^2 + 3\tau_{yz}^2}$$

Numerical data

E=100 GPa	ν=0.3	α=1.4$\cdot 10^{-5}$ $1/C^\circ$
α_c=500 J/sm^2C°	k=45 W/mC°	t=375 mm
b=375 mm	l_1=62.5 mm	l_2=100 mm
l_3=125 mm	l_4=150 mm	d=25 mm
T=800 C°	T_∞=20 C°	T_{water} = 95 C°
p=7.0 Mpa	R=100 mm	c= 466 J/kgC°
$\rho = 7800kg/m^3$		

Table 1: Numerical data

(Due to the high temperatures inside the combustion chamber, Young's modules is reduced.)

Report

A fundamental ingredient in all research is that it should be possible to regenerate the results obtained based on the report. In the present situation this implies that the appended matlab code should only be considered as supporting material. Moreover, note that one variable for grading the report is the structure of the computer code, i.e. you should choose suitable names for variables etc. A suitable structure for the report is:

- **Introduction**: Description of the problem, geometry etc. Keep this section as short as possible.
- **Procedure**: How the problems are solved (weak formulation, application of boundary conditions, thermal strains etc.). Note that you are encouraged to make references to text-books etc. It is important to carefully present all calculations that are not available in the litterature.
- **Results**: Present the results in illustrative figures and/or tables. Note that the results should be commented such that the reader can not missunderstand the results (correct labels, units, figure texts etc.)
- **Disscusion**: A disscusion of the results. You might want to discuss sources of errors and accuracy in this section.
- **Computer Code**: Note that the code should be easy to follow and all declared variables should have intuitive names and so on.

Collaboration

The task should be solved in groups of *two* or *individually*. For further details see www.solid.lth.se and navigate to the course page.