Finite element method – Nonlinear systems
FHL066 – 2013
Division of Solid Mechanics

Project 1 – General instructions

A written report including results/conclusions should be returned to the Division of Solid Mechanics no later than 28/11 10.00.

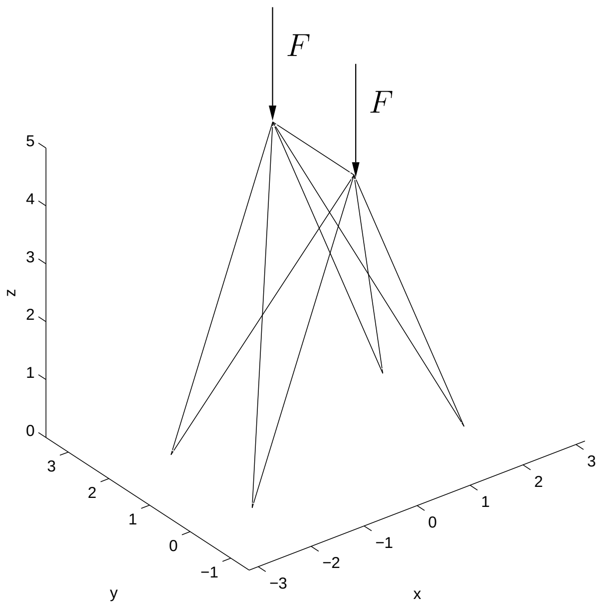
The assignment serves as part of the examination. A maximum of 10 points can be obtained. The task should be solved in groups of two or individually. If two persons work together they will obtain the same amount of points.

The assignment considers the nonlinear behavior of a structure. The task is to analyze the nonlinear response of a simple structure. To solve the problem Matlab should be used. In the toolbox Calfem, certain general FE-routines are already established and the task is to establish the extra routines needed to solve the nonlinear boundary value problem.

For the grading of the assignment, a number of questions should be adressed. The number of points obtained will be determined by how these questions are answered.

A three-dimensional arch structure

The figure below shows a arch structure that has several singular points including four bifurcation points. The arch structure consists of nine truss elements. The cross section area times the E-modulus is given as $AE = 10^4$. The element defined by the commands bar3ge, bar3gs, bar3gf should be used.



The geometry and boundary conditions can be found in the file data2013_1.m on the course home page. The boundary conditions for the trusses are applied at $z = 0$. At the two end points at $z = 5$ two forces exist, points in negative z-direction.

The assignment conists of implementing the following task.

- Write a script file newt.m containing the Newton-Raphson loop that is able to calculate the force-displacement response of the fundamental path.
- Write a script file crisf.m containing the arch-length method by Crisfield.

The second Piola Kirchhoff stress should be taken as

$$S = \frac{E}{2}\frac{(\ln(2\epsilon_G + 1))}{(2\epsilon_G + 1)^{(1/2)}} \tag{1}$$

where ϵ_G is the Lagrange strain.

For the report, the following questions should be adressed. In addition, the code should be included as an appendix.

a) Write a short introduction about the problem, including the geometry and boundary conditions.

b) Show in a plot F vs. the displacement in the z-direction, for the normal Newton-Raphson solution. Explain the behavior.

c) Explain why the introduction the constraint equation $f = 0$ is needed.

d) In figure 8.4 in [1] an interpretation of the constraint equation, f, used in the Crisfield algorithm is shown for $\psi = 1$. Draw a similar figure and interpret the constraint equation for $\psi = 0$.

e) In equation 8.10 in [1], the constraint equation $\sum_\alpha(\Delta a_\alpha - c_\alpha)^2$ is shown. If the constraint equation instead is $\sum_\alpha(\Delta a_\alpha - c_\alpha)$, would that still ensure a_α is changed by c_α in every increment?

f) Explain the main point about the differences between the algorithms present in Box 8.1 and Box 8.2 in [1]

g) Plot the force-displacement response of the fundamental path using the arc-length method by Crisfield.

h) Use the arch-length method and introduce different types of imperfections into the system such that the bifurcated paths can be located. At least two bifurcated paths should be found. Use plots of force vs. displacement in x-, y- and z-direction for illustration. Also the command eldisp3 is useful for visualizing.

[1] *The additional notes obtained during the lectures. Chapter 8.*