

# Finite element method - Nonlinear systems

## FHLN20 - 2018

### Division of Solid Mechanics

#### Project 1 - General instructions

A written report including results/conclusions should be returned to the Division of Solid Mechanics no later than **10/12 2018, 10.00**.

The assignment serves as part of the examination. A maximum of 10 points can be obtained. The task should be solved in groups of two or individually. If two persons work together, they will obtain the same amount of points.

The assignment considers an analysis of the non-linear behavior of a simple structure. To solve the problem MATLAB should be used. In the toolbox CALFEM, certain general FE-routines are already established and the task is to establish the extra routines needed to solve the non-linear boundary value problem.

The report should contain a description of the problem, the solution procedure that is needed, as well as the results from the calculations in form of illustrative figures and tables. The program codes should be well commented and included in an Appendix.

When writing the text it can be assumed that the reader has basic knowledge of Solid Mechanics, but it has been a while since he/she dealt with this type of analysis. After reading the report, the reader should be able to obtain all the relevant results just by reading through the report, i.e. without using the included program.

The report should be structured and give a professional description of the methods and the obtained results and be no longer than 10 pages (appendix excluded).

# Octet-truss lattice structure

An octet-truss structure will be considered for the analysis. The octet-truss structure is a lightweight lattice structure specifically designed to withstand axial loading. An illustrative image of the octet lattice structure and its unit cell is shown in Fig. 1. For simplicity, only a single unit cell will be considered in the analysis.

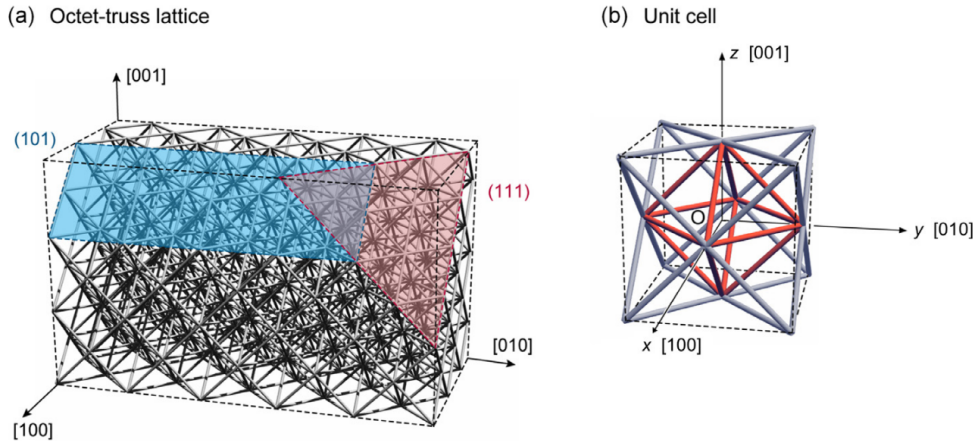


Figure 1: Octet lattice structure [1]. a) Example of a full lattice structure with multiple unit cells, b) The unit cell which periodically constructs the lattice structure, note that the coordinate system is not the same as in Fig. 2.

The unit cell should be modeled using the 3D bar elements implemented during the exercise sessions. In all calculations use should be made of the element commands `bar3ge`, `bar3gs` and `bar3gf` that have been coded during the exercise sessions.

## Geometry and boundary conditions

A MATLAB script, `data.m`, with the geometry, material properties and associated mesh is available on the web page. For convenience, in this assignment dimensionless units are used, such that  $\bar{u} = u/a$  and  $\bar{P} = P/EA$ , where  $a$  is the side length of the unit cell,  $E$  is the Young's modulus and  $A$  is the initial cross section area of the bars. This means that all sides have length  $a = 1$  and the initial length of the bars is  $l_0 = 1/\sqrt{2}$ .

The mesh of the unit-cell is depicted in Fig. 2. The unit-cell consists of 36 bar elements and 14 nodes. All the relevant properties of the mesh, such as the  $\mathbf{E}_{dis}$  and  $\mathbf{N}_{dis}$  matrices, are provided in the script `data.m`.

The nodes at the top ( $z = 1$ ) are loaded uniformly in the  $z$ -direction, the load pattern is provided by  $\mathbf{P}$  in `data.m`. The nodes at the bottom ( $z = 0$ ) are constrained to prevent

rigid body motion. The Dirichlet boundary conditions are provided by the matrix **bc** in **data.m**.

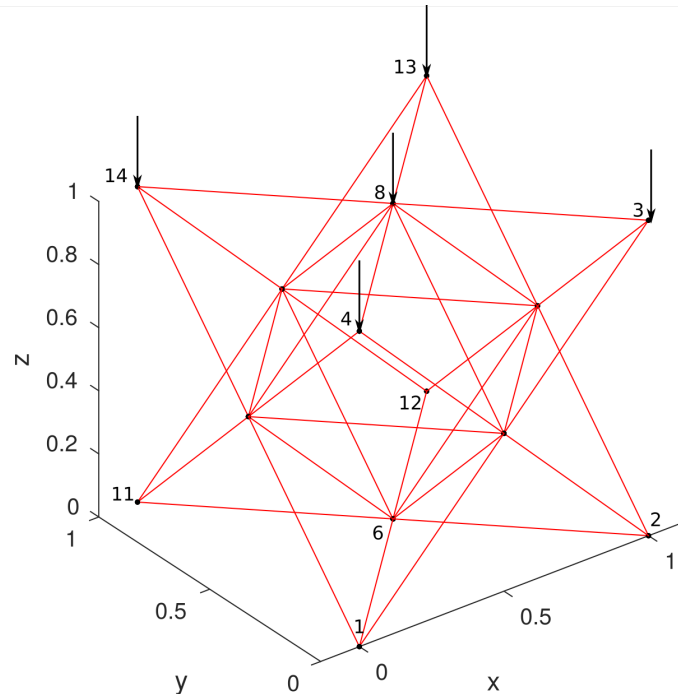


Figure 2: Meshed octet unit cell and the uniform load case.

## Path-following analysis

The tasks to be considered are

1. Write a script file **crisf.m** containing the constrained path-following method. Assume a linear relation the Green strain,  $\varepsilon_g$ , and its conjugated stress,  $\sigma_g$ . Plot the load parameter,  $\lambda$ , as a function of  $u_z^8$ , i.e. the displacement of node 8 in the  $z$ -direction. Also plot  $u_x^8$  and  $u_z^8$ , i.e. the displacement of node 8 in the  $x$ -direction and  $z$ -direction, in relation to each other.
2. Assume that the material is described by

$$w(\Lambda) = \frac{E}{5} \left( \frac{1}{\Lambda} + \frac{1}{4}\Lambda^4 \right) \quad (1)$$

where  $E$  is the same Young's modulus as previously and  $\Lambda = \frac{l}{l_0}$  is the stretch. Write a script file **crifn.m** using the non-linear material model and the constrained path-following method. Perform the same calculations as in task 1 and compare the results.

3. Introduce at least two perturbations, one in the geometry and one in the load such that the equilibrium curve branching from the main path becomes visible. Use the linear material law and compare the results with those obtained from task 1.

The manual to the toolbox CALFEM is available on the web page. Commands such as `eldisp3`, `eldraw3` and `drawnow` are useful for visualization.

## References

- [1] V Dong, L. Deshpande and H Wadley. Mechanical response of Ti-6Al-4V octet-truss lattice structures. *International Journal of Solids and Structures*, 60-61:107–124, 2015.