

Finite element method – Nonlinear systems
FHL066 – 2015
Division of Solid Mechanics

Project 1 – General instructions

A written report including results/conclusions should be returned to the Division of Solid Mechanics no later than 2015 1/12, 10.00.

The assignment serves as part of the examination. A maximum of 10 points can be obtained. The task should be solved in groups of two or individually. If two persons work together they will obtain the same amount of points.

The assignment considers an analysis of the nonlinear behavior of a simple structure. To solve the problem Matlab should be used. In the toolbox Calfem, certain general FE-routines are already established and the task is to establish the extra routines needed to solve the nonlinear boundary value problem.

The report should contain a description of the problem, the solution procedure that is needed as well as the results from the calculations in form of illustrative figures and tables. The program codes should be well commented and included in an Appendix.

When writing the text it can be assumed that the reader has basic knowledge of Solid Mechanics, but it has been a while since he/she dealt with this type of analysis. After reading the report, the reader should be able to obtain all the relevant results just by reading through the report, i.e. without using the included program.

The report should be structured and give a professional description of the methods and the obtained results and be no longer than 10 pages (appendix excluded).

A triangular truss structure

The shallow truss structure, Fig. 1, shows a complex behavior when loaded and is dominated with various types of bifurcations. The geometry is given in Fig. 1, assuming $E = 10 \text{ MPa}$, $A = 1 \text{ mm}^2$, $L = 100 \text{ mm}$, $h_1 = 20 \text{ mm}$, $h_2 = 30 \text{ mm}$, .

In the calculations the element commands `bar3ge`, `bar3gs` and `bar3gf` should be used, that have been coded during the exercise sessions.

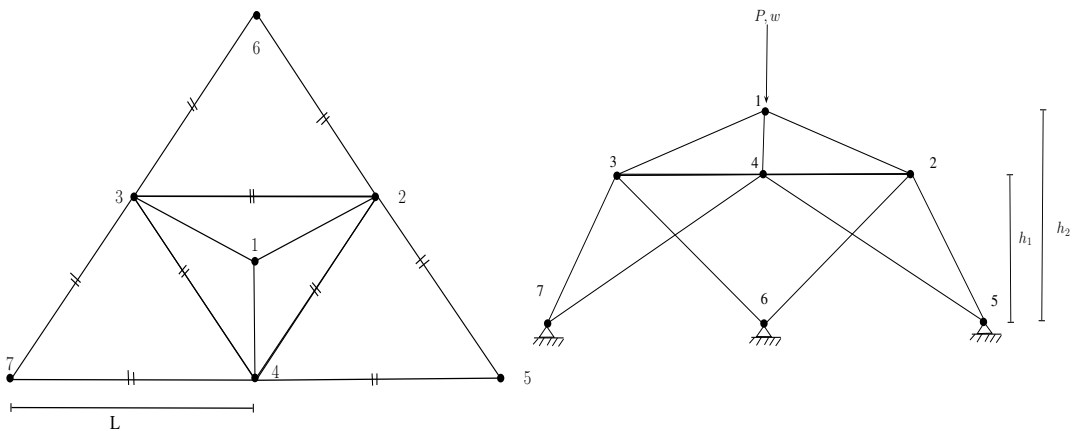


Figure 1: Geometrical properties of the triangular truss structure. The bars with (//) are of equal length.

The task to be considered are

- Assume the structure is at equilibrium at load step n . The residual at load step $n + 1$ is defined as $\mathbf{r}_{n+1} = \mathbf{f}_{int}^{n+1} - \mathbf{f}_{ext}^{n+1}$, where \mathbf{f}_{int}^{n+1} are the internal forces at the degrees of freedom of the structure and \mathbf{f}_{ext}^{n+1} are the external forces. Perform a truncated Taylor expansion of the residual and show how the displacements \mathbf{a}_{n+1} can be obtained through an iteration scheme. No explicit matrix expression are needed. Identify the tangent stiffness matrix \mathbf{K} .
- Consider the manual `bar3ge` and `bar3gf` given on the course homepage. Using eq (2.21) and eq (2.22) from Krenk (2009), show that the internal force at the degrees of freedom of a bar element can be written as

$$\mathbf{f}_{int}^{e,n+1} = \frac{AS}{l_0} \begin{bmatrix} (x_1 - x_2) + (a_1 - a_4) \\ (y_1 - y_2) + (a_2 - a_5) \\ (z_1 - z_2) + (a_3 - a_6) \\ (x_2 - x_1) + (a_4 - a_1) \\ (y_2 - y_1) + (a_5 - a_2) \\ (z_2 - z_1) + (a_6 - a_3) \end{bmatrix} \quad (1)$$

where $a_1 \dots a_6$ are the nodal displacements, $x_1, x_2, y_1, y_2, z_1, z_2$ are the reference nodal coordinates, A the cross-section area, l_0 the initial length and S the second Piola Kirchoff stress.

- Using (1), derive the expression of the tangential stiffness matrix for a bar element used in the Newton-Raphson iteration scheme.
- Write a script file `newt.m` containing the Newton-Raphson loop. Force controlled loading is applied. Assume a linear relation between the second Piola-Kirchhoff stress and Green's strain, i.e. $S = E\epsilon_G$. Plot the applied load P as function of the displacement of node 1 i z-direction, i.e w_1 .
- Write a script file `crisf.m` containing the arch-length method by Crisfield. Apply load P until the structure is completely 'snapped-through'. Plot the applied load P as function of the displacement of node 1 in the z-direction, i.e w_1 .

- Assume that the material is described by

$$S = \frac{E}{2} \frac{\ln(2\epsilon_G + 1)}{2(2\epsilon_G + 1)^{1/2}}$$

Write a script file `crifn.m` using the nonlinear material model when Crisfields method is used. Plot the applied load P as function of the displacement of node 1 in the z-direction, i.e w_1 .

- Introduce at least two perturbations, one in the geometry and one in the load such that different equilibrium curves branching from the main path becomes visible.

Commands such as `eldisp3`, `eldraw3` and `drawnow` are useful for visualization.