# Finite element method – Nonlinear systems
# FHLN20 – 2017
# Division of Solid Mechanics

## Project 1 – General instructions

A written report including results/conclusions should be returned to the Division of Solid Mechanics no later than **4/12 2017, 10.00**.

The assignment serves as part of the examination. A maximum of 10 points can be obtained. The task should be solved in groups of two or individually. If two persons work together they will obtain the same amount of points.

The assignment considers an analysis of the nonlinear behavior of a simple structure. To solve the problem Matlab should be used. In the toolbox Calfem, certain general FE-routines are already established and the task is to establish the extra routines needed to solve the nonlinear boundary value problem.

The report should contain a description of the problem, the solution procedure that is needed as well as the results from the calculations in form of illustrative figures and tables. The program codes should be well commented and included in an Appendix.

When writing the text it can be assumed that the reader has basic knowledge of Solid Mechanics, but it has been a while since he/she dealt with this type of analysis. After reading the report, the reader should be able to obtain all the relevant results just by reading through the report, i.e. without using the included program.

The report should be structured and give a professional description of the methods and the obtained results and be no longer than 10 pages (appendix excluded).

# Truss structures

Two different truss structures will be considered, both revealing complex behavior when loaded and dominated by various types of bifurcations. First a simple plane three-member truss structure will be consider, then as a final task a sixteen-member shallow truss structure will be considered.

In all calculations use should be made of the element commands bar3ge, bar3gs and bar3gf, that have been coded during the exercise sessions.

## Three-member truss structure

The geometry is given in Fig. 1. Initially when no load is applied the angle $\varphi = \pi/3$, the length of the bars is $l_o = 1$m. For the stiffness the ratio

$$\beta = \frac{E'A'}{EA} \tag{1}$$

is useful. Select a stiffness $EA$, where $E$ and $A$ are the Young's modulus and initial cross section area. Use $\beta$ to control the stiffness of the upper bar. In the presentation of the results the normalized load

$$\lambda = \frac{F}{E'A'} \tag{2}$$

should be used. The problem is two-dimensional.

The tasks to be considered are

1. Write a script file newt.m containing the Newton-Raphson loop. Force controlled loading is applied. Assume a linear relation between the second Piola-Kirchhoff stress and Green's strain. Select a model value for $E$. Plot the normalized applied load as function of the displacement for $\beta = 10$.

2. Write a script file crisf.m containing the arch-length method by Crisfield. Plot $\lambda$ as a function of $u$ for different values of $\beta$, i.e. make the upper bar less stiff. Consider only the symmetric equilibrium path. Comment upon the results when the upper bar becomes less stiff, can the result be explained.

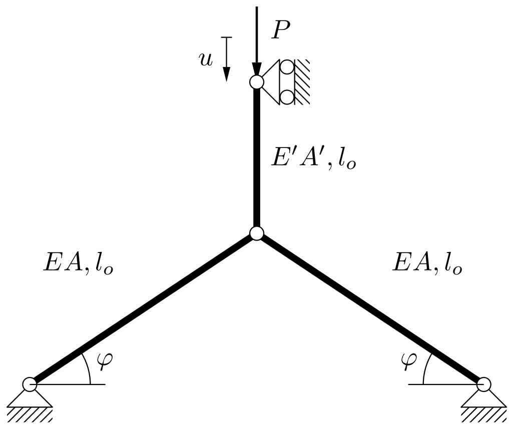


Figure 1: Illustration of the simple truss system.

3. Assume that the material is described by

$$S = \frac{E}{3}\left(\Lambda - \frac{1}{\Lambda^2}\right) \tag{3}$$

where $E$ is the same Young's modulus as previously and $\Lambda = \frac{l}{l_0}$ is the stretch. Write a script file crifn.m using the nonlinear material model when Crisfields method is used. Perform the same calculations as was done in (2) and compare the results.

4. Introduce at least two perturbations, one in the geometry and one in the load such that the equilibrium curve branching from the main path becomes visible.

Commands such as *eldisp3*, *eldraw3* and *drawnow* are useful for visualization.

## Sixteen-member shallow truss structure

The truss structure is illustrated in Fig. 2, and more details are given in the article [1]. Necessary data should be found in the article.

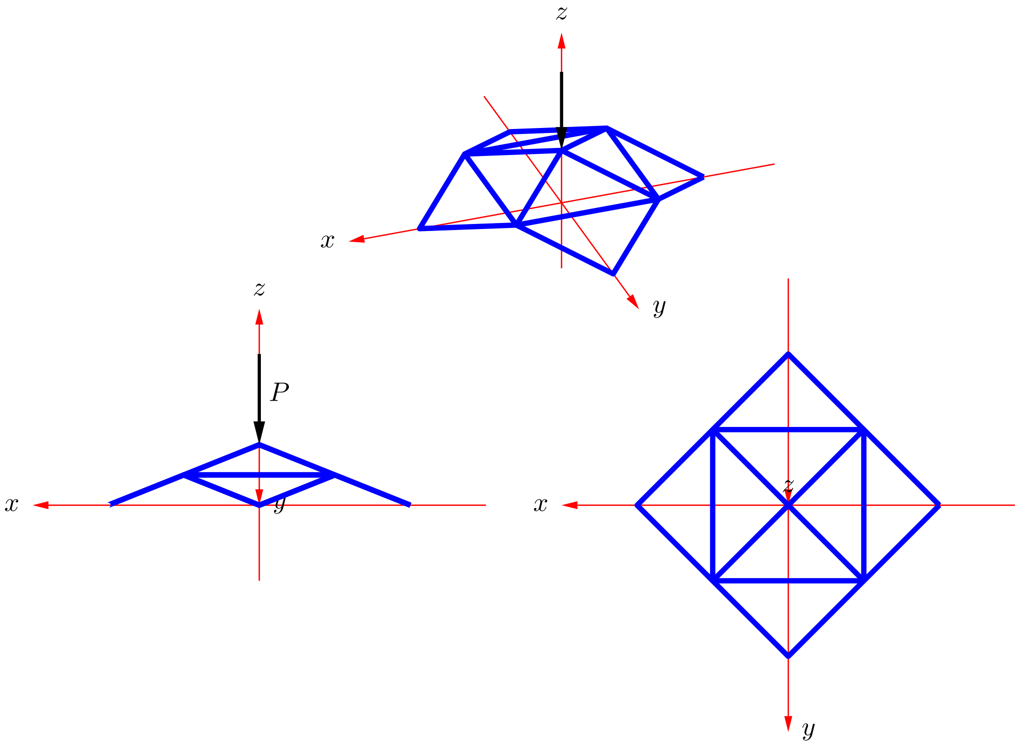


Figure 2: Illustration of the sixteen-member truss structure

The following tasks should be considered

1. Modify the Crisfield's method in the previous example, denote the main program crisfs.m. Consider the loading given in the article, obtain a plot similar to Fig. 13 (in the article). Show this for both when a linear relation is assumed between the second Piola-Kirchhoff stress and Green's strain as well as when the non-linear stress strain law given by (3) is used.

2. Introduce different types of imperfections into the system such that bifurcated paths can be located. At least two bifurcated paths should be found. These should be clearly illustrated in graphs, where the equilibrium path deviate from the symmetric main equilibrium path. In addition plots showing the equilibrium path and the behaviour of the structure after the bifurcation, i.e. continued loading.

[1] G. A. Hrinda, "Snap-trough instability patterns in truss structures", 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th, 12 - 15 April 2010, Orlando, Florida.