Finite element method – Nonlinear systems
FHL066 – 2014
Division of Solid Mechanics

Project 2 – General instructions

A written report including results/conclusions should be returned to the Division of Solid Mechanics no later than 2015 20/1, 10.00.

The assignment serves as part of the examination. A maximum of 20 points can be obtained. The task should be solved in groups of two or individually. If two persons work together they will obtain the same amount of points.

The assignment considers an analysis of the nonlinear behavior of a simple structure. To solve the problem Matlab should be used. In the toolbox Calfem, certain general FE-routines are already established and the task is to establish the extra routines needed to solve the nonlinear boundary value problem.

The report should contain a description of the problem, the solution procedure that is needed as well as the results from the calculations in form of illustrative figures and tables. The program codes should be well commented and included in an Appendix.

When writing the text it can be assumed that the reader has basic knowledge of Solid Mechanics, but it has been a while since he/she dealt with this type of analysis. After reading the report, the reader should be able to obtain all the relevant results just by reading through the report, i.e. without using the included program.

The report should be structured and give a professional description of the methods and the obtained results and be no longer than 20 pages (appendix excluded).

Static analysis of a continuum

A L-shaped bar with rectangular cross section will be analysed, as shown in Fig 1.

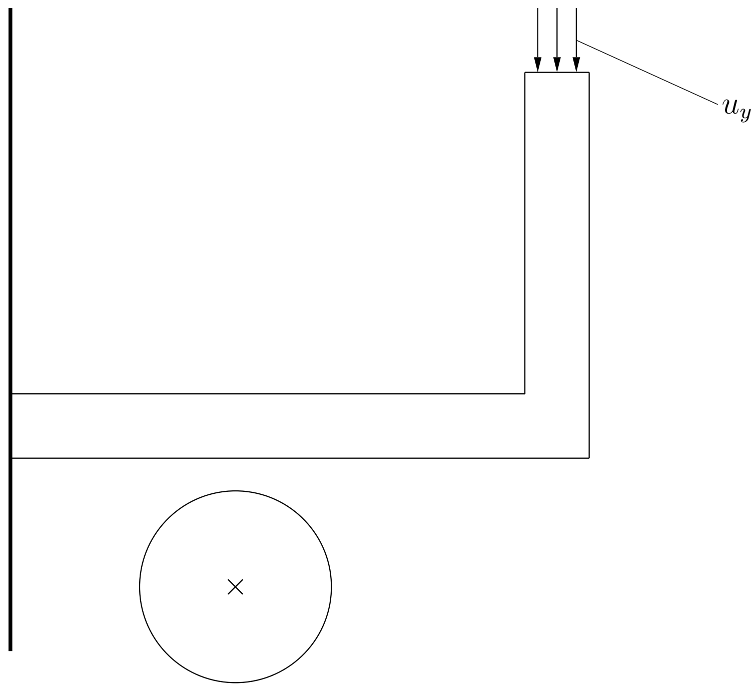

Figure 1: A L-shaped bar subjected to a displacement u_y. A rigid cylinder limits the deformation of the bar.

The first task is to solve the boundary value problem without introducing the rigid cylinder and in the second task, the cylinder will be included. The material is assumed to be described by the strain energy function

$$U = \frac{1}{2}K[\frac{1}{2}(J^2 - 1) - \ln J] + \frac{1}{2}G(J^{-2/3}\mathrm{tr}(\boldsymbol{C}) - 3) \tag{1}$$

where K and G are the initial bulk and shear moduli, respectively, these can be obtained from the elastic modulus $E = 10$ GPa and Poissons ratio

$\nu = 0.35$. Moreover, $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$ where $\boldsymbol{F}$ is the deformation gradient and $J = \det \boldsymbol{F}$.

For simplicity plane strain conditions are assumed, i.e. thickness can be set to 1 mm. The radius of the cylinder is 5 mm, the geometry provided in the matlab file `static2014_1.mat`.

In the static numerical solution procedures a total Lagrangian formulation should be used with 4-node isoparametric elements. Loading and boundary conditions are provided in the binary geometry files. Displacement loading is assumed and applied at the top boundary directed in negative y-direction indicated in Fig. 1.

1) Solve the boundary value problem using a static algorithm without introducing the cylinder. Plot the deformation of the structure and the force vs displacement of the top boundary indicated in the Fig. 1. Load the structure until a displacement of $u_y = 40$ mm is achieved.

2) The contact of the L-shaped bar with the cylinder will now be considered. The problem is a simplified model of the penalty method used in FE-codes. For this purpose the three dimensional bar elements defined in chapter 2 in Krenk [1] with a special constitutive model will be used.

The constitutive model used is given by

$$N = \begin{cases} \frac{k}{\Lambda}(\Lambda - \Lambda_c) & \text{if} \quad \Lambda < \Lambda_c \\ 0 & \text{if} \quad \Lambda \geq \Lambda_c \end{cases} \tag{2}$$

where

$$\Lambda = \sqrt{2\epsilon_G + 1} = \frac{l}{l_0} \qquad \Lambda_c = \frac{r}{l_0} \tag{3}$$

The strain ϵ_G is the usual Green strain and Λ is the stretch, i.e. equation (2) replaces equation (2.20) in Krenk (2009). The length r is the radius of the cylinder as shown in Fig. 1. A suitable value for k should be determined, such that the penetrations through the cylinder becomes minimum.

Write two functions. The first one calculating the normal force N, as

$$N = \texttt{norfb(ec,ee,k,r)} \tag{4}$$

according to equation (2). The input arguments are defined in the manual pages for the bar3g element, the additional parameters are defined in equation (2).

The second function should calculate the material stiffness, i.e $D = dN/d\epsilon_G$ by

$$D = \texttt{bstiff(ec,ee,k,r)} \tag{5}$$

The bar elements are now located such that one end is located at the center of cylinder, the other end is connected to a node in the structure. Then when the length of the bar becomes less than the radius of the cylinder a contact force will be present.This is described by the constitutive law (2).The geometry is given in `static2014_2.mat`.

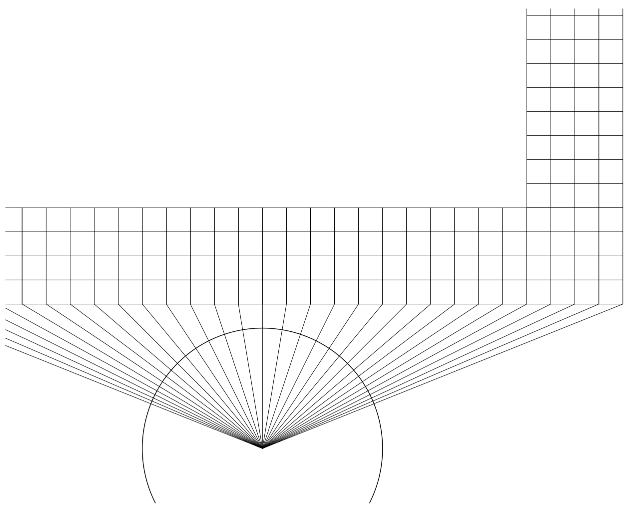

Figure 2: The contact bars connected to the nodes of the elements.

Solve the boundary value problem using a static algorithm including the contact of the cylinder. Plot the deformation of the structure and the force vs displacement of the top boundary indicated in the Fig. 1. Load the structure until a displacement of $u_y = 40$ mm is achieved. Compare the simulated response with that obtained in 1).

Dynamic analysis of a bouncing ball

A dynamic analysis of a ball bouncing against the wall located at $y = 0$ will be considered, see Fig. 3.

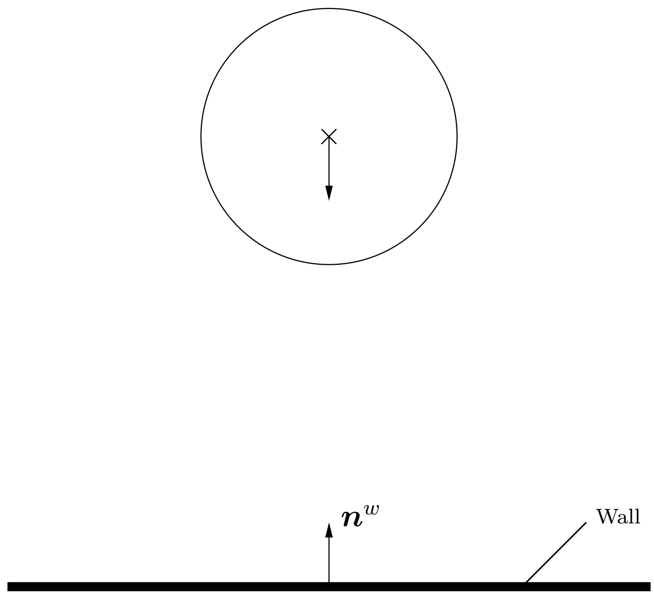

Figure 3: A ball influenced by gravitational forces dropped from rest, towards the wall located a $y = 0$.

It is assumed that the ball is only influenced by body forces due to gravitation forces, $\boldsymbol{f}_b$. The ball is initially at rest and dropped a distance 30 cm from the wall.

The initial density is assumed to equal 100kg$/m^3$. Note that if [mm] is used in the calculations a correct scaling of ρ is needed such that $\rho \ddot{u} V = [N]$ where

V is the volume. The material response is here assumed to be described by the St. Venant-Kirchhoff model, i.e. the strain energy is given by

$$U = \frac{1}{2}\boldsymbol{E}^T\boldsymbol{D}\boldsymbol{E} \tag{6}$$

where $\boldsymbol{D}$ is the constant elasticity matrix for plane strain conditions. The elastic properties are the same as in the static analysis.

A linear damping matrix $\boldsymbol{C}_d$ should be included. It is assumed to be given as fraction of the mass matrix, i.e.

$$\boldsymbol{C}_d = \alpha\boldsymbol{M} \tag{7}$$

where α is a scalar and $\boldsymbol{M}$ is the mass matrix.

The contact of the wall will be modeled using a node-based penalty formulation. This is introduced by including a force vector $\boldsymbol{f}_{cont}$ to the residual, i.e.

$$\boldsymbol{r} = (\boldsymbol{f}_{int} + \boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}}) - (\boldsymbol{f}_b + \boldsymbol{f}_{cont}) \tag{8}$$

The contact force vector is assumed to be given by

$$\boldsymbol{f}_{contact} = \sum_{i=1}^{n_c} \bar{\epsilon}_N \bar{g}_{Ni} \boldsymbol{n}_i^w \tag{9}$$

where $\boldsymbol{n}^w$ is the normal vector to the wall $\bar{\epsilon}_N$ is a penalty parameter and $\bar{g}_{Ni}$ is the penetration function, defined by

$$\bar{g}_{Ni} = \begin{cases} (\boldsymbol{x}^i - \boldsymbol{x}^w)^T\boldsymbol{n}^w & \text{if} \qquad (\boldsymbol{x}^i - \boldsymbol{x}^w)^T\boldsymbol{n}^w < 0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $\boldsymbol{x}^i$ is the nodal position along the boundary of the ball and $\boldsymbol{x}^w$ is the closet point projection, i.e. the point on the wall which gives has the smallest distance to the node $\boldsymbol{x}^i$. Note that a parameterisation of the wall can be written as $\boldsymbol{x}_t^w = (t, 0)$ where $t \in \mathbb{R}$.

1) Show that the linearisation of the contact force vector $\boldsymbol{f}_{contact}$ provides the stiffness matrix given by

$$\boldsymbol{K}_{contact} = \sum_{i=1}^{n_c} \epsilon_N \boldsymbol{n}^w (\boldsymbol{n}^w)^T \tag{11}$$

2) Solve the dynamic boundary value problem using the Newmark method. Using the newmark paramaters $\gamma = 0.5$ and $\beta = 0.25$ and the damping coefficient $\alpha = 2.5$ find a suitable value for $\bar{\epsilon}_N$ (less than 0) in the Newmark algorithm which gives a convergent solution. The thickness can be set to 0.2 m. Plot the displacement of the ball vs time. The geometry of the ball is given in `dynamic2014.m`. The variable `NodesBoundary` contains the global numbering of the nodes on the boundary (corresponding to the row number of `coord`), the variable `dofBoundary` contains the numbering of the corresponding degree of freedoms for the node in `NodesBoundary` and the xy coordinates of the nodes on the boundary is given in `coordB`.

3) Plot the variation of the energy during the process. Consider different time step lengths, comment upon the results obtained. For this purpose a function

`[KinE, IntE] = plan3gEd(` suitable arguments`)` (12)

should be written.

4) Vary some parameters in the model, such as the mass, damping coefficient α, elastic modulus and $\bar{\epsilon}_N$ etc, to determine how the bouncing capacity of the ball changes. That is, define a suitable measure for a bouncing coefficient and determine what variables set the size of the parameter.

Hints

To speed up the program you should instead of `zeros` use `sparse` to initialize matrices.

The `assem` command in Calfem is very slow, a slightly faster algorithm is obtained by using

```
nd = Edof(element, 2:nrdof _ element);
K(nd,nd) = K(nd,nd) + Ke
```