HÅLLFASTHETSLÄRA, LTH

Examination in computational materials modeling

TID: 2015-01-10, kl 8.00-13.00

Maximalt 60 poäng kan erhållas på denna tenta. För godkänt krävs 30 poäng.

Tillåtet hjälpmedel: räknare

Uppgift nr	1	2	3	4	5	6
Besvarad (sätt x)						
Poäng						

NAMN:____

PERSONNUMMER:____________________ÅRSKURS: ____________

PROBLEM 1 (10p.)

For a linear elastic material the stress response $\sigma_{ij} = s_{ij} + \frac{1}{3}\delta_{ij}\sigma_{kk}$ can be described by the relations

$$\sigma_{kk} = 3K\epsilon_{kk}$$
$$s_{ij} = 2Ge_{ij}$$

where the total strain is $\epsilon_{kl} = e_{kl} + \frac{1}{3}\delta_{kl}\epsilon_{pp}$ and

$$G = \frac{E}{2(1+\nu)}, \qquad K = \frac{E}{3(1-2\nu)}$$

a) Assume that a strain energy function $W(\tilde{I}_1, \tilde{J}_2, \tilde{J}_3)$ exists, where

$$\tilde{I}_1 = \epsilon_{kk}, \qquad \tilde{J}_2 = \frac{1}{2}e_{ij}e_{ji}, \qquad \tilde{J}_3 = \frac{1}{3}e_{ik}e_{kl}e_{li}$$

Derive the most general stress strain response from

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}}$$

b) Based on the result in a) identify the specific format for the strain energy considered in this example.

c) Assuming that the strain energy is positive, i.e. $W > 0$, identity the constraints on E and ν.

d) For the linear elastic material derive the material stiffness D_{ijkl} defined by the relation $\sigma_{ij} = D_{ijkl}\epsilon_{kl}$.

PROBLEM 2 (10p.)

Derive the incremental stress strain relation in elasto-plasticity. Make use of that the strain can be decomposed as

$$\epsilon_{ij} = \epsilon^e_{ij} + \epsilon^p_{ij}$$

where superscipt e denotes the elastic part and superscipt p the plastic part of the strains. The Hooke's law is given as

$$\sigma_{ij} = D_{ijkl}\epsilon^e_{kl} \tag{1}$$

The evolution laws are defined as

$$\dot{\epsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}, \qquad \dot{\kappa} = \dot{\lambda} k, \qquad \dot{\lambda} \geq 0$$

where $f(\sigma_{ij}, K)$ is the yield surface, $K = K(\kappa)$ is the hardening function and $k = k(\sigma_{ij}, K)$ is related to the hardening of the material.

a) Assume plastic loading, use the consistency condition to derive

$$\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}\dot{\epsilon}_{kl} - \dot{\lambda} A = 0$$

and identify A.

b) Use the consistency condition in the above relations to derive the strain driven incremental law

$$\dot{\sigma}_{ij} = D_{ijkl}^{ep}\dot{\epsilon}_{kl}$$

Name one situation where the above relation breaks down.

PROBLEM 3 (10p.)

For materials with a special type of tetragonal crystal structure only 7 material parameters are needed to describe the linear relation between stresses and strains.

To derive the constitutive relation for tetragonal symmetry we will consider a rotation of the coordinate system in the $x_1 - x_2$ plane by $\pi/2$ which is described by the transformation matrix

$$[A_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Recall that the transformation of second-order tensors, i.e. stresses and strains are given by

$$\sigma'_{ij} = A_{ik}\sigma_{kl}A_{jl} \qquad \epsilon'_{ij} = A_{ik}\epsilon_{kl}A_{jl}$$

The general stress- strain relation is given by

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{13} \\ 2\epsilon_{23} \end{bmatrix}$$

a) Derive the transformation of the different stress and strain components if transformation matrix A_{ij} is used.

b) Use the arguments about elastic symmetry planes and the transformation matrix stated above show how the $\boldsymbol{D}$-matrix reduces. Only the two first stresses σ_{11} and σ_{22} need to be considered.

PROBLEM 4 (10p.)

For uniaxial experimental tests, i.e. uniaxial loading

$$[\sigma_{ij}] = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

on a metal it was found that a good fitting could be obtained by using a power law format for the stress strain relation, given by

$$\sigma = \begin{cases} E\epsilon & \text{when} \quad \epsilon \leq \frac{\sigma_{yo}}{E} \\ \sigma_{yo} + k\sigma_{yo}(\epsilon^p)^n & \text{when} \quad \epsilon \geq \frac{\sigma_{yo}}{E} \end{cases}$$

where σ, ϵ and ϵ^p are the uniaxial stress, strain and plastic strain, respectively. Moreover, E is Young's modulus and σ_{yo} is the initial yield stress, and finally n and k are material parameters.

Assuming that the material can be described by a kinematic hardening von Mises model during plastic loading

$$f = \left(\frac{3}{2}\bar{s}_{ij}\bar{s}_{ij}\right)^{1/2} - \sigma_{yo} = 0$$

where $\bar{s}_{ij} = s_{ij} - \alpha_{ij}$ is the reduced deviatoric stress tensor, α_{ij} is a deviatoric back-stress tensor and $s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$ is the deviatoric stress tensor Associated plasticity is assumed, i.e

$$\dot{\epsilon}^p_{ij} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}, \qquad \dot{\lambda} \geq 0$$

For evolution of the back-stress it is assumed that Melan-prager's evolution law is valid

$$\dot{\alpha}_{ij} = c\dot{\epsilon}^p_{ij}$$

where c is a positive material parameter that may depend on the load history.

a) Calculations the plastic strain rate for the above model.

b) For a general load case and the choice strain hardening, i.e.

$$\dot{\kappa} = \dot{\epsilon}_{eff} = \left(\frac{2}{3}\dot{\epsilon}^p_{ij}\dot{\epsilon}^p_{ij}\right)^{1/2}$$

identify the rate of internal variable by use of the flow rule for plastic strain rates.

c) Considering uniaxial loading identify the choice of internal variable (i.e. κ).

d) For uniaxial loading identify the form of $\dot{\alpha}_{ij}$

e) Given that uniaxial loading, given that the experimental data can be fitted to the power law function, identify $c(\kappa)$.

PROBLEM 5 (10p.)

Let failure of a material be defined by the Drucker-Prager criterion

$$F(\sigma_{ij}) = \sqrt{3J_2} + \alpha I_1 - \beta = 0 \tag{1}$$

where

$$J_2 = \frac{1}{2}s_{ij}s_{ij}, \qquad I_1 = \sigma_{kk}, \qquad s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

Moreover, α and β are material parameters.

a) Illustrate (1) in the deviatoric plane and in the meridian plane, i.e. the $I_1 - \sqrt{J_2}$ plane. (Note that $I_1 = \sigma_{kk}$). A principal sketch is sufficient.

b) Derive the uniaxial tension and uniaxial compression paths in terms of I_1 and $\sqrt{J_2}$. Draw the stress paths in the deviatoric plane and in the meridian plane.

c) Reduce the Drucker-Prager criterion to plane stress conditions, i.e. $\sigma_3 = 0$.

d) Determine the uniaxial tensile strength σ_t, uniaxial compression strength σ_c, biaxial tensile strength σ_{bt} and the biaxial compressive strength σ_{bc} in terms of α and β.

e) Draw the shape of the Drucker-Prager criterion in the $\sigma_1 - \sigma_2$ plane, and mark the locations of the stresses in problem d).

PROBLEM 6 (10p.)

In a certain experiment it is found that yielding of a material occurs under the following states of principal stresses

$$(\sigma_1,\, \sigma_2,\, \sigma_3) = (20,\, 0,\, 5)$$
$$(\sigma_1,\, \sigma_2,\, \sigma_3) = (-1,\, 0,\, 18)$$

Assuming that the material is isotropic, that the hydrostatic stress does not affect the yielding and the yield stress is the same in tension and compression.

Plot as many points as you can derive from these observations in the $\sigma_1 - \sigma_2$ space, i.e. biaxial loading, where $\sigma_3 = 0$ (plane stress).