HÅLLFASTHETSLÄRA, LTH

Examination in computational materials modeling

TID: 2016-28-10, kl 14.00-19.00

Maximalt 60 poäng kan erhållas på denna tenta. För godkänt krävs 30 poäng.

Tillåtet hjälpmedel: räknare

PROBLEM 1 (10p.)

For hyper-elasticity, the stresses σ_{pq} are given by

$$\sigma_{pq} = \frac{\partial W}{\partial \epsilon^e_{pq}} \tag{1}$$

where W is the strain energy and ϵ^e_{pq} are the elastic strains. We have

$$\epsilon^e_{kl} = \epsilon_{kl} - \epsilon^p_{kl}$$

where ϵ_{kl} and ϵ^p_{kl} denote the total strains and the plastic strains, respectively. For an elasto-plastic material the strain energy is a function of the elastic strains, i.e. $W = W(\epsilon^e_{ij})$.

a) Derive the general stress-strain relation using (1) for an isotropic material assuming that $W = W(\tilde{I}^e_1, \tilde{I}^e_2, \tilde{I}^e_3)$ with the invariants

$$\tilde{I}^e_1 = \epsilon^e_{kk} \qquad \tilde{I}^e_2 = \frac{1}{2}\,\epsilon^e_{ij}\epsilon^e_{ji} \qquad \tilde{I}^e_3 = \frac{1}{3}\,\epsilon^e_{ij}\epsilon^e_{jk}\epsilon^e_{ki} \tag{2}$$

b) For isotropic linear elasticity the strain energy is given by

$$W = \frac{1}{2}\lambda\,(\tilde{I}^e_1)^2 + 2\mu\,\tilde{I}^e_2 \tag{3}$$

where λ and μ are material constants. Show that the constitutive relation is given by

$$\sigma_{ij} = \lambda\epsilon^e_{kk}\delta_{ij} + 2\mu\epsilon^e_{ij} \tag{4}$$

c) Show that (4) can be written as

$$\sigma_{ij} = D_{ijkl}\epsilon^e_{kl} \tag{5}$$

where

$$D_{ijkl} = 2G[\frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \frac{\nu}{1-2\nu}\delta_{ij}\delta_{kl}]$$

The constant material parameters are related via

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad \mu = G = \frac{E}{2(1+\nu)} \tag{6}$$

PROBLEM 2 (10p.)

Equations of motion are given by

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = \varrho \ddot{u}_i \tag{1}$$

where σ_{ij} is the stress tensor, b_i the body force vector, ϱ the density and $\ddot{u}_i$ denotes the acceleration vector.

a) From (1) derive the weak formulation.

b) Introduce the approximation $\boldsymbol{u} = \boldsymbol{N}\boldsymbol{a}$, where $\boldsymbol{u}$ is the displacement vector, $\boldsymbol{N}$ the global shape functions and $\boldsymbol{a}$ the nodal displacement vector. Make use Galerkin's method and from the weak formulation derive the corresponding finite element formulation.

PROBLEM 3 (10p.)

For initial yielding, the criteria of von Mises and Tresca are given by

$$\sqrt{3J_2} - \sigma_{yo} = 0 \quad ; \qquad \sigma_1 - \sigma_3 - \sigma_{yo} = 0$$

respectively. Here

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \quad ; \qquad s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

Note that in Tresca's criterion, we have

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

where tension is considered as a positive quantity.

Consider the stress state

$$\sigma_{ij} = \begin{bmatrix} 0 & \sigma_{12} & 0 \\ \sigma_{21} & 0 & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

a) For the given stress state, determine the yield condition according to von Mises.

b) For the given stress state, determine the yield condition according to Tresca. Assume that $\sigma_{33} < 0$ and $|\sigma_{33}| > |\sigma_{12}|$.

PROBLEM 4 (10p.)

The orthotropic initial yield criteria of Hill, independent of hydrostatic stress, can be written as

$$F(s_{11} - s_{22})^2 + G(s_{11} - s_{33})^2 + H(s_{22} - s_{33})^2 + 2Ls_{12}^2 + 2Ms_{13}^2 + 2Ns_{23}^2 - 1 = 0$$

where s_{ij} denotes deviatoric stresses, and F, G, H, L, M and N are material parameters.

a) Show that the above yield criteria can be written as

$$\boldsymbol{s}^T \boldsymbol{P} \boldsymbol{s} - 1 = 0 \qquad (1)$$

identify $\boldsymbol{s}$ containing the deviatoric stresses and $\boldsymbol{P}$ containing the material parameters.

b) Propose one way how the model can be calibrated, determine also the material parameters in Hill's model based on the proposed calibration procedure. The equation system containing the material parameters should be established but need not to be solved.

PROBLEM 5 (10p.)

A von Mises material is considered

$$f(\sigma_{ij}, K) = (\frac{3}{2} s_{kl} s_{kl})^{1/2} - \sigma_{y0} = 0 \qquad (1)$$

where $s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$ and σ_{y0} = initial yield stress. The associated flow rule states that

$$\dot{\epsilon}_{ij}^p = \dot{\lambda} \frac{\partial f}{\partial \sigma_{ij}} \qquad (2)$$

and the effective plastic strain rate is defined by

$$\dot{\epsilon}_{eff}^p = (\frac{2}{3} \dot{\epsilon}_{ij}^p \dot{\epsilon}_{ij}^p)^{1/2} \qquad (3)$$

Isotropic elasticity is assumed, such that the volumetric part, ϵ_{kk}^e, and the deviatoric part, $e_{ij}^e = \epsilon_{ij}^e - \frac{1}{3}\epsilon_{kk}^e \delta_{ij}$ of the elastic strain tensor, ϵ_{ij}^e, are given by

$$\epsilon_{kk}^e = \frac{1}{3K^e} \sigma_{kk} \qquad e_{ij}^e = \frac{1}{2G^e} s_{ij} \qquad (4)$$

where K^e denotes the bulk modulus and and G^e the shear modulus. The total strain is given by $\epsilon_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p$.

Assume proportional loading, i.e.

$$\sigma_{ij} = \beta(t)\sigma_{ij}^* \tag{5}$$

where σ_{ij}^* is a fixed and constant stress state and the scalar function β which is a function of time, t, controls the load level.

Consider proportional loading and show that the above plasticity model can be written as a nonlinear isotropic Hooke formulation, given by

$$\sigma_{kk} = 3K\epsilon_{kk} \qquad s_{ij} = 2Ge_{ij} \tag{6}$$

Identify K and G.

PROBLEM 6 (10p.)

Consider the postulate of maximum dissipation

$$(\sigma_{ij} - \sigma_{ij}^*)\dot{\epsilon}_{ij}^p \geq 0 \tag{1}$$

where σ_{ij} is some stress on the yield surface, i.e. $f(\sigma_{ij}, K) = 0$, whereas σ_{ij}^* denotes any stress state that fulfills $f(\sigma_{ij}^*, K) \leq 0$.

a) Excluding the trivial solution, $\epsilon_{ij}^p = 0$ and considering a fixed value of the hardening parameter K. Show that from (1), the yield function f can not be concave. Use figure (a) below for your argumentation.

b) Use figure (b) below to show the normality principle (associated plasticity) holds.

a) part of concave yield function — b) part of convex yield function