HÅLLFASTHETSLÄRA, LTH

Examination in computational materials modeling

TID: 2019-11-01, kl 14.00-19.00

Maximalt 60 poäng kan erhållas på denna tenta. För godkänt krävs 30 poäng.

Tillåtet hjälpmedel: räknare

Uppgift nr	1	2	3	4	5	6
Besvarad (sätt x)						
Poäng						

Anonymkod:______________________________

Personlig identifierare:__________________

PROBLEM 1 (10p.)

A hyper-elastic model for describing the stress-strain response for arterial tissue has been proposed by Delfino (1997). For small strains the strain energy function can be written as

$$W = W_v + W_d \tag{1}$$

where

$$W_v = \frac{1}{2} K \tilde{I}_1^2 \quad W_d = \frac{a}{b} \left\{ \exp(\frac{b}{2} \tilde{J}_2) - 1 \right\} \tag{2}$$

In (2) K is the bulk modulus and the two other material parameters are $a = 44.2$kPa and $b = 16.7$ which is a non-dimensional parameter. Moreover, the two invariants given in (2) are defined as

$$\tilde{I}_1 = \epsilon_{kk} \quad \text{and} \quad \tilde{J}_2 = \frac{1}{2} e_{ij} e_{ij}$$

The deviatoric strain tensor is defined from $\epsilon_{ij} = e_{ij} + 1/3 \delta_{ij} \epsilon_{kk}$.

a) Derive the stress-strain relation using

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}}$$

b) Establish the secant stiffness D^s_{ijkl} defined from $\sigma_{ij} = D^s_{ijkl} \epsilon_{kl}$, as well as the tangent stiffness D^t_{ijkl} defined from $d\sigma_{ij} = D^t_{ijkl} d\epsilon_{kl}$

PROBLEM 2 (10p.)

The Rousselier (1987) yield criterion for porous material with a void volume fraction denoted by f is given by

$$F = \frac{\sigma_{eff}}{(1-f)\sigma_y(\epsilon^p_{eff})} + \frac{\sigma_o}{\sigma_y(\epsilon^p_{eff})} Df \exp\left(\frac{\sigma_{kk}}{(1-f)\sigma_o} \right) - 1 = 0$$

is often used to model porous metallic material. The current yield stress is given by $\sigma_y(\epsilon^p_{eff})$, where

$$\dot{\epsilon}^p_{eff} = (\frac{2}{3} \dot{\epsilon}^p_{ij} \dot{\epsilon}^p_{ij})^{1/2}$$

is the rate of the effective plastic strain, and ϵ^p_{ij} is the plastic strain.

Moreover D and σ_o are two (constant) material parameters. Furthermore the effective stress is defined as

$$\sigma_{eff} = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$$

a) Assume in-plane pure shear i.e.

$$[\sigma_{ij}] = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Calculate the stress τ of which yielding starts.

b) Show that if associated plasticity holds i.e.

$$\dot{\epsilon}_{ij}^{p} = \dot{\lambda} \frac{\partial F}{\partial \sigma_{ij}}$$

the plastic strains in a general loading situation will contribute to the volume change.

PROBLEM 3 (10p.)

Explain how the following models will behave during uniaxial cyclic loading. A graphical explanation (illustration) showing the response in a uniaxial state and its corresponding response in the deviatoric plane is necessary. Explain in words what happens to the elastic region.

a) Isotropic hardening von Mises model.

b) Kinematic hardening von Mises model.

c) Mixed hardening von Mises model.

Consider the three different stress states

$$\boldsymbol{\sigma}_1 = \begin{bmatrix} 2\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} \quad \boldsymbol{\sigma}_2 = \begin{bmatrix} \sigma & \tau & 0 \\ \tau & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} \quad \boldsymbol{\sigma}_3 = \begin{bmatrix} -2\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

In the deviatoric plane determine the direction of the loading, i.e. the angle θ given by

$$\cos(3\theta) = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$$

where

$$J_2 = \frac{1}{2}s_{ij}s_{ji}, \qquad J_3 = \frac{1}{3}s_{ik}s_{kj}s_{ji}$$

and $s_{ij} = \sigma_{ij} - 1/3\sigma_{kk}\delta_{ij}$ denotes the deviatoric stress tensor and σ_{ij} is the stress tensor.

d) Calculate the angle corresponding to the three stresses.

e) Assume the von Mises yield criterion is valid for initial yielding, i.e.

$$F(\sigma_{ij}) = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{1/2} - \sigma_{yo} = 0$$

Calculate the value of τ in $\boldsymbol{\sigma}_2$ for which yielding starts.

PROBLEM 4 (10p.)

Let yield and failure of a material be defined by the Drucker-Prager criterion

$$f(\sigma_{ij}) = \sqrt{3J_2} + \alpha I_1 - \beta = 0 \tag{1}$$

where

$$J_2 = \frac{1}{2}s_{ij}s_{ij}, \qquad I_1 = \sigma_{kk}, \qquad s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

Moreover, β is a material parameters and $\alpha = \alpha(\epsilon_d^p)$, where

$$\dot{\epsilon}_d^p = (\dot{e}_{ij}^p \dot{e}_{ij}^p)^{1/2}$$

Assume that initial value and the value are failure is given by

$$\alpha_i = \alpha(0), \qquad \alpha_f = \alpha(\epsilon_d^p \to \infty)$$

where $\alpha_i < \alpha_f$

a) Illustrate (1) in the deviatoric plane and in the meridian plane, i.e. the $I_1 - \sqrt{3J_2}$ plane, consider both α_i and α_f. A sketch is sufficient.

b) Derive the uniaxial tension and uniaxial compression paths in terms of I_1 and $\sqrt{J_2}$. Include both the initial yield surface and the failure surface. Draw the stress paths in the deviatoric plane and in the meridian plane such that it reaches the failure surface.

c) Reduce the Drucker-Prager criterion to plane stress conditions, i.e. $\sigma_3 = 0$.

d) Determine the initial: uniaxial tensile strength σ_t, uniaxial compression strength σ_c, biaxial tensile strength σ_{bt} and the biaxial compressive strength σ_{bc} in terms of α and β.

e) Draw the shape of the Drucker-Prager criterion in the $\sigma_1 - \sigma_2$ plane, and mark the locations of the stresses in problem d).

PROBLEM 5 (10p.)

Consider the Tsai-Wu criterion that is commonly used as a fracture criterion for composites, paper and paperboard, i.e.

$$F = \boldsymbol{\sigma}^T \boldsymbol{P} \boldsymbol{\sigma} + \boldsymbol{q}^T \boldsymbol{\sigma} - 1 = 0$$

where

$$\boldsymbol{\sigma}^T = \begin{bmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{13} & \sigma_{23} \end{bmatrix}$$

Moreover, for a coordinate system aligned with the material directions

$$\boldsymbol{P} = \begin{bmatrix} A & -F & -G & 0 & 0 & 0 \\ -F & B & -H & 0 & 0 & 0 \\ -G & -H & C & 0 & 0 & 0 \\ 0 & 0 & 0 & 2L & 0 & 0 \\ 0 & 0 & 0 & 0 & 2M & 0 \\ 0 & 0 & 0 & 0 & 0 & 2N \end{bmatrix}$$

and

$$\boldsymbol{q}^T = \begin{bmatrix} q_1 & q_2 & q_3 & 0 & 0 & 0 \end{bmatrix}$$

The task is to calibrate the model.

a) Consider different uniaxial loading conditions (both tension as well as compression) show that A, B, C, q_1, q_2 and q_3 can be determined. Derive only an explicit expression for A and q_1.

b) Show that if a biaxial loading condition is applied, i.e. $\sigma_{11} \neq 0$ and $\sigma_{22} \neq 0$ otherwise $\sigma_{ij} = 0$ that F can be determined.

PROBLEM 6 (10p.)

For materials with a special type of tetragonal crystal structure only 7 material parameters are needed to describe the linear relation between stresses and strains.

To derive the constitutive relation for tetragonal symmetry we will consider a rotation of the coordinate system in the $x_1 - x_2$ plane by $\pi/2$ which is described by the transformation matrix

$$[A_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Recall that the transformation of second-order tensors, i.e. stresses and strains are given by

$$\sigma'_{ij} = A_{ik}\sigma_{kl}A_{jl} \qquad \epsilon'_{ij} = A_{ik}\epsilon_{kl}A_{jl}$$

The general stress- strain relation is given by

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{13} \\ 2\epsilon_{23} \end{bmatrix}$$

a) Derive the transformation of the different stress and strain components if transformation matrix A_{ij} is used.

b) Use the arguments about elastic symmetry planes and the transformation matrix stated above show how the $\boldsymbol{D}$-matrix reduces. Only the two stresses σ_{33} and σ_{12} need to be considered.