HÅLLFASTHETSLÄRA, LTH

Examination in computational materials modeling

TID: 2012-10-24, kl 14.00-19.00

Maximalt 50 poäng kan erhållas på denna dugga. För godkänt krävs 25 poäng.

Tillåtet hjälpmedel: räknare

Uppgift nr	1	2	3	4	5
Besvarad (sätt x)					
Poäng					

NAMN:____

PERSONNUMMER:____________________ ÅRSKURS: ____________

PROBLEM 1 (10p.)

Derive the incremental strain-stress relation in plasticity. Make use of

$$\epsilon_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p$$

and that the elastic strains are given by

$$\sigma_{ij} = D_{ijkl}\epsilon_{kl}^e$$

where C_{ijkl} is the constant flexibility tensor. The evolution laws

$$\dot{\epsilon}_{ij}^p = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}} \qquad \dot{\kappa} = \dot{\lambda}k \qquad \dot{\lambda} \geq 0$$

where $f(\sigma_{ij}, K)$ is the yield function, $K = K(\kappa)$ is the hardening function and $k = k(\sigma_{ij}, K)$ is related to the hardening of the material.

a) Based on the consistency condition $\dot{f} = 0$, show that

$$\dot{\lambda} = \frac{1}{A}\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}\dot{\epsilon}_{kl}$$

and identify A

b) Identify the incremental relation

$$\dot{\sigma}_{ij} = D_{ijkl}^{ep}\dot{\epsilon}_{kl}$$

i.e. derive the D_{ijkl}^{ep}.

PROBLEM 2 (10p.)

The von Mises yield function for kinematic hardening is given by

$$f = \bar{\sigma}_{eff} - \sigma_{yo}$$

where

$$\bar{\sigma}_{eff} = \left(\frac{3}{2}\bar{s}_{ij}\bar{s}_{ij}\right)^{1/2}, \qquad \bar{s}_{ij} = s_{ij} - \alpha_{ij}$$

where s_{ij} is the deviatoric stress tensor and α_{ij} the back-stress tensor.

Assume that the evolution law for kinematic hardening is given by the Armstrong-Frederick model, i.e.

$$\dot{\alpha}_{ij} = h\left(\frac{2}{3}\dot{\epsilon}_{ij}^{p} - \frac{\alpha_{ij}}{\alpha_\infty}\dot{\epsilon}_{eff}^{p}\right) \tag{1}$$

where h and α_∞ are constants and

$$\dot{\epsilon}_{eff}^{p} = \left(\frac{2}{3}\dot{\epsilon}_{ij}^{p}\dot{\epsilon}_{ij}^{p}\right)^{1/2}$$

where the plastic strain rate is derived from associated plasticity.

a) Calculate the gradients

$$\frac{\partial f}{\partial \sigma_{ij}} \quad \text{and} \quad \frac{\partial f}{\partial \alpha_{ij}}$$

b) From the consistency condition and using Armstrong-Fredericks model derive the expression for the generalized hardening modulus H given in

$$\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} - H\dot{\lambda} = 0$$

c) Illustrate graphically in the deviatoric plane the situation of load reversal after a loading into the plastic region and explain how the generalized hardening modulus will change. Make use of the result derived in b).

d) Shown that the consistency condition can be written as

$$\dot{\epsilon}_{ij}^{p}\dot{\sigma}_{ij} = H(\dot{\epsilon}_{eff}^{p})^2$$

and provide a graphical interpretation of H for the uniaxial loading situation.

e) For uniaxial loading situation and considering the Armstrong Frederick model derive $H = H(\epsilon_{eff}^{p})$

Hint: First solve α from (1), then use $f = 0$ to solve H.

PROBLEM 3 (10p.)

The initial yield criterion of Drucker-Prager is defined by

$$f = \sqrt{3J_2} + \alpha I_1 - \beta = 0$$

where α and β are parameters and

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \qquad s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk} \qquad I_1 = \sigma_{kk}$$

Consider the stress state

$$[\sigma_{ij}] = \begin{bmatrix} \sigma & 0 & \tau \\ 0 & 0 & 0 \\ \tau & 0 & 0 \end{bmatrix}$$

a) Assume that loading takes place such that $\tau = \sigma$. Calculate the value of σ for which yielding starts. Both $\sigma > 0$ and $\sigma < 0$ should be considered.

b) Draw the shape of the Drucker-Prager yield criterion in the meridian plane, $\sqrt{3J_2} - I_1$ and the deviatoric plane.

c) Draw the loading path $\tau = \sigma$ in the meridian plane, consider both $\sigma > 0$ and $\sigma < 0$.

d) In the deviatoric plane draw the loading path $\sigma = 0$ and $\tau \neq 0$. (one path is sufficient). Hint: the angle is given by

$$\cos(3\theta) = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} \quad \text{where} \quad J_3 = \frac{1}{3} s_{ik} s_{kj} s_{ji}$$

PROBLEM 4 (10p.)

For hyper-elasticity the strains are given by

$$\epsilon_{ij} = \frac{\partial C}{\partial \sigma_{ij}} \tag{1}$$

where C is the complementary energy. For isotropic materials we have $C = C(I_1, J_2)$ where the invariants are defined as

$$I_1 = \sigma_{kk} \qquad J_2 = \frac{1}{2} s_{ij} s_{ij}$$

where $s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$.

a) For (1) and assuming isotropic material, derive the most general form for isotropic hyper-elasticity.

b) To model elasticity of soils the following form of the complementary energy is used

$$C = -\frac{(1-2\nu)}{6Mp_a}\frac{p_a^{2\lambda}}{(\lambda-1)}\frac{1}{(I_1^2+RJ_2)^{(\lambda-1)}}$$

where

$$R = \frac{6(1+\nu)}{1-2\nu}$$

and p_a is a scaling constant, M and λ are constant dimensionless material parameters and ν is Poisson's ratio. Use the result in a) to derive the stress-strain law for this material. (The model was introduced by Lade and Nelson (1987))

c) Use the result in b) to obtain the following from of the stress-strain law

$$\epsilon_{ij} = C_{ijkl}\sigma_{kl}$$

Symmetry properties should be preserved.

PROBLEM 5 (10p.)

The orthotropic yield criterion by Hoffman can be written as

$$f = \boldsymbol{\sigma}^T \boldsymbol{P}\boldsymbol{\sigma} + \boldsymbol{q}^T\boldsymbol{\sigma} - 1 = 0$$

where

$$\boldsymbol{\sigma}^T = [\sigma_{11},\, \sigma_{22},\, \sigma_{33},\, \sigma_{12},\, \sigma_{23},\, \sigma_{13}\,]$$

and $\boldsymbol{P}$ is square and symmetric matrix and $\boldsymbol{q}$ is a column matrix.

a) Let the x_i'-coordinate system denote the mirror image of the x_i-coordinate system in symmetry plane. Assume that $\mathrm{x}_1 - x_2$ and $x_1 - x_3$ defines two different symmetry planes. Define the two corresponding $\boldsymbol{L}$ matrices defining the mapping of second order tensor between coordinate systems x_i and x_i'.

Hint: Let σ_{ij} denote the components of a second-order tensor in the x_i-coordinate system, its components σ'_{ij} in the x'_i-coordinate system is given by

$$\sigma'_{ij} = A_{ik}\sigma_{kl}A_{jl} = A_{ik}A_{jl}\sigma_{kl}$$

which in matrix format can be written as

$$\boldsymbol{\sigma}' = \boldsymbol{L\sigma}$$

and $\boldsymbol{A}^T$ is defined by three column matrices, which are the base vectors of the x'_i-coordinate system given in x_i-coordinates.

b) With the starting point that both $\boldsymbol{P}$ and $\boldsymbol{q}$ are fully populated. Make use of symmetry planes to derive the reduced forms of $\boldsymbol{P}$ and $\boldsymbol{q}$. Assume $x_1 - x_2$ and that $x_1 - x_3$ defines two different symmetry planes.

Hint: Use the results in a)

c) Derive an orthotropic pressure independent version of the Hoffman yield criterion. Derived the resulting $\boldsymbol{P}$ and $\boldsymbol{q}$.

Hint: Make use of that the stress matrix can decomposed into a deviatoric part and a pressure part.