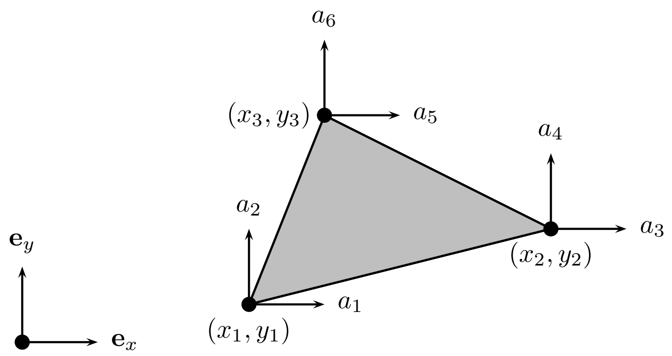
**Purpose:**

Compute the algorithmic tangent stiffness matrix for a triangular 3 node element under plane stress conditions.




**Syntax:**

Dats=alg_tan_stiff(sigma,Dstar,ep_eff,dlambda,mp)

**Description:**

alg_tan_stiff provides the algorithmic tangent stiffness matrix $\mathsf{D}_{ats}$ for a triangular 3 node element. The in-plane stresses are provided by sigma;

$$\text{sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}$$

Dstar is the linear elastic material tangent for plane stress case. ep_eff is the effective plastic strains $\varepsilon^p_{eff}$, dlambda is the increment $\Delta\lambda$ and mp a vector containing the material parameters needed;

$$\text{mp}^T = \begin{bmatrix} E & \nu & k & n & \sigma_{y0} \end{bmatrix}$$

The algorithmic tangent stiffness is defined according to equation (18.64) in the course book;

$$\mathbf{D}_{ats} = \mathbf{D}^a - \frac{1}{A^a}\mathbf{D}^a \frac{\partial f}{\partial \boldsymbol{\sigma}} \left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \mathbf{D}^a$$

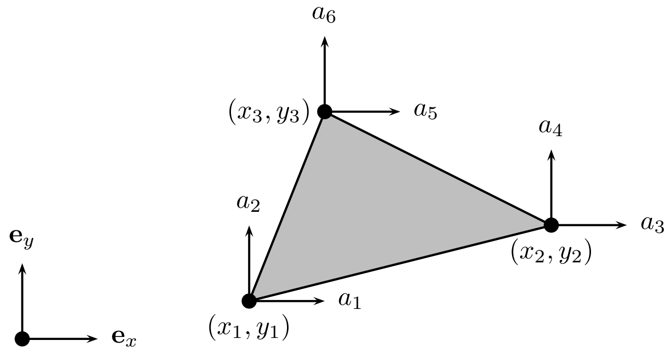
where

$$\mathbf{D}^a = \left(\mathbf{D}*^{-1} + \Delta\lambda \frac{\partial^2 f}{\partial \boldsymbol{\sigma} \partial \boldsymbol{\sigma}}\right)^{-1}, \quad A^a = \left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \mathbf{D}^a \frac{\partial f}{\partial \boldsymbol{\sigma}} - \frac{\partial f}{\partial K} d^a$$

$\mathbf{D}*$ denotes the linear elastic material tangent given by Dstar. If the response is elastic alg_tan_stiff returns Dats=Dstar.

**Purpose:**

Check for elasto-plastic response in a triangular 3 node element under plane stress conditions and update variables accordingly.




**Syntax:**

[sigma,ep_eff,dlambda]=update_variables(sigmaEq,delta_eps,ep_effEq,Dstar,mp)

**Description:**

update_variables provides the following updates;

sigma - in-plane stresses $\boldsymbol{\sigma}^T = \begin{bmatrix}\sigma_{11} & \sigma_{22} & \sigma_{12}\end{bmatrix}$

dlambda - increment $\Delta\lambda$

ep_eff - effective plastic strain $\varepsilon^p_{eff}$ using the radial return method for isotropic von Mises hardening plasticity.

The variables are computed with help of the stress and effective plastic strain from the last equilibrium state, sigmaEq and ep_effEq respectively and the increment in strains between the last equilibrium state and the current; delta_eps.

The increment $\Delta\lambda$ needed to update the stresses and strains are also computed and could be used as a indicator for plasticity later on in the program and will therefore also be used as output from the function.

Moreover Dstar denotes the linear elastic material tangent and mp is a vector containing the material parameters;

$$\mathsf{mp}^T = \begin{bmatrix}E & \nu & k & n & \sigma_{y0}\end{bmatrix}$$