GENERALIZED PLASTIC MODULUS

We have

$$f(\sigma_{ij}, K^\alpha) = 0 \qquad \text{at plastic loading}$$

Consistency

$$\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} + \frac{\partial f}{\partial K^\alpha}\dot{K}^\alpha = 0$$

But $K^\alpha = K^\alpha(\kappa^\beta)$

$$\Rightarrow \quad \frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} + \frac{\partial f}{\partial K^\alpha}\frac{\partial K^\alpha}{\partial \kappa^\beta}\dot{\kappa}^\beta = 0$$

Evolution laws $\dot{\kappa}^\beta = \dot{\lambda} k^\beta(\sigma_{ij}, K^\alpha)$

$$\Rightarrow \quad \frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} + \underbrace{\frac{\partial f}{\partial K^\alpha}\frac{\partial K^\alpha}{\partial \kappa^\beta}k^\beta}_{-H}\dot{\lambda} = 0$$

Define generalized plastic modulus

$$\boxed{H = -\frac{\partial f}{\partial K^\alpha}\frac{\partial K^\alpha}{\partial \kappa^\beta}k^\beta}$$

Consistency

$$\boxed{\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} - H\dot{\lambda} = 0}$$

STRAIN DRIVEN FORMAT

We found

$$\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} - \dot{\lambda}H = 0$$

Hooke's law

$$\dot{\sigma}_{kl} = D_{ijkl}(\dot{\epsilon}_{kl} - \dot{\epsilon}^p_{kl})$$

Flow rule

$$\dot{\epsilon}^p_{ij} = \dot{\lambda}\frac{\partial g}{\partial \sigma_{ij}} \quad \Rightarrow \quad \dot{\sigma}_{kl} = D_{ijkl}(\dot{\epsilon}_{kl} - \dot{\lambda}\frac{\partial g}{\partial \sigma_{kl}})$$

i.e.

$$\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\epsilon}_{kl} - \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\frac{\partial g}{\partial \sigma_{kl}} - \dot{\lambda}H = 0$$

The plastic multiplier

$$\boxed{\dot{\lambda} = \frac{1}{A}\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\epsilon}_{kl}}$$

where

$$A = H + \frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\frac{\partial g}{\partial \sigma_{kl}} > 0$$

Incremental stress-strain law

$$\boxed{\dot{\sigma}_{ij} = D^{ep}_{ijkl}\dot{\epsilon}_{kl}}$$

where

$$D^{ep}_{ijkl} = D_{ijkl} - \frac{1}{A}D_{ijmn}\frac{\partial g}{\partial \sigma_{mn}}\frac{\partial f}{\partial \sigma_{pq}}D_{pqkl}$$

INTERPRETATION OF H

We found

$$\dot{\sigma}_{ij} = D^{ep}_{ijkl}\dot{\epsilon}_{kl}$$

where

$$D^{ep}_{ijkl} = D_{ijkl} - \frac{1}{A} D_{ijmn} \frac{\partial g}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} D_{pqkl}$$

$$A = H + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial g}{\partial \sigma_{kl}} > 0$$

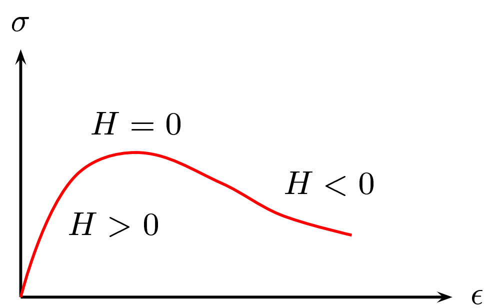
We known that

$$H = 0 \quad \Rightarrow \quad \text{ideal plasticity}$$

$$D^{ep}_{ijkl} \rightarrow \infty \qquad \text{if} \quad A \rightarrow \infty \quad \text{i.e.} \quad H \rightarrow \infty$$

i.e.

$$H > 0 \quad \Rightarrow \quad \text{hardening plasticity}$$
$$H < 0 \quad \Rightarrow \quad \text{softening plasticity}$$



GENERAL LOADING/UNLOADING CRITERIA

We found

$$\dot{\lambda} = \frac{1}{A} \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}\dot{\epsilon}_{kl} \qquad \dot{\lambda} > 0 \qquad A > 0$$

Define

$$\dot{\sigma}^{e}_{ij} = D_{ijkl}\dot{\epsilon}_{kl} \qquad \Rightarrow \qquad \dot{\lambda} = \frac{1}{A} \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}^{e}_{ij}$$

If

$$f < 0 \quad \Rightarrow \quad \text{elastic response}$$

If

$$f = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}^{e}_{ij} > 0 \quad \Rightarrow \quad \text{plastic response}$$
$$f = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}^{e}_{ij} = 0 \quad \Rightarrow \quad \text{neutral response}$$
$$f = 0 \quad \text{and} \quad \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}^{e}_{ij} < 0 \quad \Rightarrow \quad \text{elastic response}$$