HÅLLFASTHETSLÄRA, LTH

Examination in computational materials modeling

TID: 2013-10-21, kl 14.00-19.00

Maximalt 60 poäng kan erhållas på denna dugga. För godkänt krävs 30 poäng.

Tillåtet hjälpmedel: räknare

Uppgift nr	1	2	3	4	5
Besvarad (sätt x)					
Poäng					

NAMN:____

PERSONNUMMER:____________________ÅRSKURS: ____________

PROBLEM 1 (12p.)

The complementary strain energy per unit volume C for linear isotropic elasticity is defined by

$$C = \frac{1+\nu}{E} I_2 - \frac{\nu}{2E} I_1^2, \qquad I_1 = \sigma_{kk}, \qquad I_2 = \frac{1}{2}\sigma_{ij}\sigma_{ij} \tag{1}$$

For linear elasticity, we may note that $W = C$ where W is the strain energy per unit volume.

The deviatoric stresses s_{ij} are defined by

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

where σ_{kk} is the hydrostatic stress.

a) Determine the strain-stress relation based on

$$\epsilon_{kl} = \frac{\partial C}{\partial \sigma_{kl}}$$

b) Using (1), show that it may be written in the form

$$C = C_d + C_h$$

where C_d is related to the deviatoric stresses and C_h is related to the hydrostatic stress.

PROBLEM 2 (12p.)

From experimental tests a non-linear elastic law was found to have the following form

$$\epsilon_{ij} = C_{ijkl}\sigma_{kl} \tag{1}$$

where

$$C_{ijkl} = \frac{(1+\nu)}{E(I_1, J_2)} \left\{ \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - \frac{\nu}{1+\nu}\delta_{ij}\delta_{kl} \right\} \tag{2}$$

Poisson's ratio ν was found to be constant, whereas Young's modulus took the following form

$$E(I_1, J_2) = E_o \left\{ \left(\frac{I_1}{p_a}\right)^2 + R\frac{J_2}{p_a^2} \right\}^\lambda \tag{3}$$

Here E_o, p_a and λ are constants and

$$R = \frac{6(1+\nu)}{1-2\nu}$$

The invariants are defined as

$$I_1 = \sigma_{kk} \qquad J_2 = \frac{1}{2}s_{ij}s_{ij}$$

where $s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$.

a) From (1) identify α_1 and α_2 in the relation

$$\epsilon_{ij} = \alpha_1\delta_{ij} + \alpha_2 s_{ij} \tag{4}$$

b) Let $C = C(I_1, J_2)$ be the complementary strain energy per unit volume show that

$$\alpha_1 = \frac{\partial C}{\partial I_1} \qquad \alpha_2 = \frac{\partial C}{\partial J_2} \tag{5}$$

based on

$$\epsilon_{ij} = \frac{\partial C}{\partial \sigma_{ij}}$$

c) What is the conditions enforced on α_1 and α_2 for a material to be hyper-elastic.

d) Show that the material in (1) - (3) is hyper-elastic.

PROBLEM 3 (12p.)

In plasticity it is assumed that the strain can be decomposed into two parts

$$\epsilon_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p \tag{1}$$

Moreover, defining Hooke's law and a hardening law as

$$\epsilon_{ij}^e = C_{ijkl}\sigma_{kl} \quad , \qquad K^\alpha = K^\alpha(\kappa^\beta) \tag{2}$$

together with the flow rule and evolution laws

$$\dot{\epsilon}_{ij}^p = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}} \quad , \qquad \dot{\kappa}^\beta = \dot{\lambda}k^\beta(\sigma_{ij}, K^\alpha) \tag{3}$$

and that during plastic loading $f(\sigma_{ij}, K^\alpha) = 0$ summarizes what is called associated plasticity.

a) Assume plastic loading, use the consistency condition to derive

$$\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} - \dot{\lambda}H = 0$$

and identify H.

b) Show in the deviatoric plane when f is given by von Mises yield function, $f = \sqrt{3J_2} - \sigma_y$, what $H > 0$, $H = 0$ and $H < 0$ describes.

c) Use the consistency condition in the above relations to derive the incremental law

$$\dot{\epsilon}_{ij} = C_{ijkl}^{ep}\dot{\sigma}_{kl}$$

Name one situation where the above relation breaks down.

PROBLEM 4 (12p.)

For general loading situations isotropic hardening of a von Mises material is given by

$$f(\sigma_{ij}, K) = \sqrt{3J_2} - \sigma_{yo} - K \leq 0 \tag{1}$$

where σ_{yo} is the initial yield stress and K is the hardening function and

$$J_2 = \frac{1}{2}s_{ij}s_{ij} \quad , \qquad s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

The associated flow rule provides

$$\dot{\epsilon}^p_{ij} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}} \quad , \qquad \dot{\lambda} \geq 0 \tag{2}$$

Consider now plane stress conditions where one principal stress $\sigma_3 = 0$ when answering the questions

a) Derive an explicit form of (1) in term of stresses, i.e. σ_{ij}, for plane stress conditions.

b) Sketch the shape of the von Mises yield surface in the principal stress space, i.e. where plane stress conditions exist

c) Sketch the shape of the von Mises yield surface for a situation of combined loading of normal stress and shear stress.

d) Use equation (2) and a) to calculate the plastic strain rates

$$\begin{bmatrix} \dot{\epsilon}^p_{11} \\ \dot{\epsilon}^p_{22} \\ \dot{\epsilon}^p_{12} \end{bmatrix}$$

PROBLEM 5 (12p.)

For materials with a crystal structure such as iron, magnesium, lithium and more only 3 material parameters are needed to describe the linear relation between stresses and strains.

To derive the constitutive relation for cubic symmetry three different transformations are used. Here we will only consider one, rotation of the coordinate system in the $x_1 - x_2$ plane by $\pi/2$ which is described by the transformation matrix

$$[A_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Recall that the transformation of second-order tensors, i.e. stresses and strains are given by

$$\sigma'_{ij} = A_{ik}\sigma_{kl}A_{jl} \qquad \epsilon'_{ij} = A_{ik}\epsilon_{kl}A_{jl}$$

Given that for orthotropy the stress- strain relation is given by

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \end{bmatrix}$$

a) Use the arguments about elastic symmetry planes and the transformation matrix stated above. Show that by considering the constitutive law for σ'_{11} the following must hold

$$D_{11} = D_{22} \quad \text{and} \quad D_{13} = D_{23}$$

b) As above show that by considering σ'_{23} that

$$D_{55} = D_{66}$$