THE FINITE ELEMENT METHOD 2018
Div. of Solid Mechanics

EXAMINATION: 2018-05-28

A maximum of 60 points can be achieved in this examination. To pass at least 30 points are required.

Permitted aid: Pocket calculator.

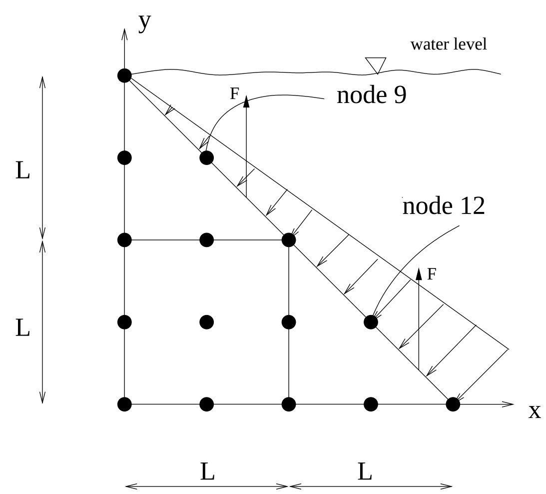
Anonymous Code ....

Course Code (FHLF01/FHLF10) ....

Personal Identifier ....

Hand in the exam!!!!!

Problem 1: (12p)



A wall is discretized using two 6 node triangular elements and one 9-node quadrilateral element. The wall is subject to two concentrated loads F applied at (x,y)=(3L/4,5L/4) and (x,y)=(7L/4,L/4). The water pressure is perpendicular to the boundary and given by $p = \rho g h$, where h represents the depth.

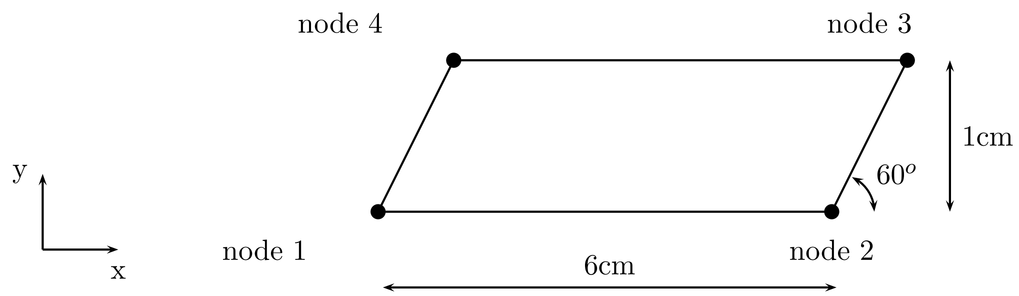
The boundary force vector in two-dimensional elasticity is given by

$$\boldsymbol{f}_b = \int_{\mathcal{L}} \boldsymbol{N}^T \boldsymbol{t} d\mathcal{L}$$

Determine the x and y components of the boundary vector, $\boldsymbol{f}_b$, in node 12.

Problem 2: (12p)

The four node element depicted below will be used in a thermal anaysis.



a) Calculate the Jacobian matrix used in isoparametric mapping.

b) Explain when and why isoparametric are needed.

Problem 3: (12p)

A two dimensional boundary value problem is governed by

$$\frac{\partial}{\partial x}\left((1+x^2)\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left((1+y^2)\frac{\partial\varphi}{\partial y}\right)+\xi=\dot{\varphi}$$

where $\xi = \xi(x, y)$ is time independent.

a) Derive the weak form corresponding to the differential equation above.

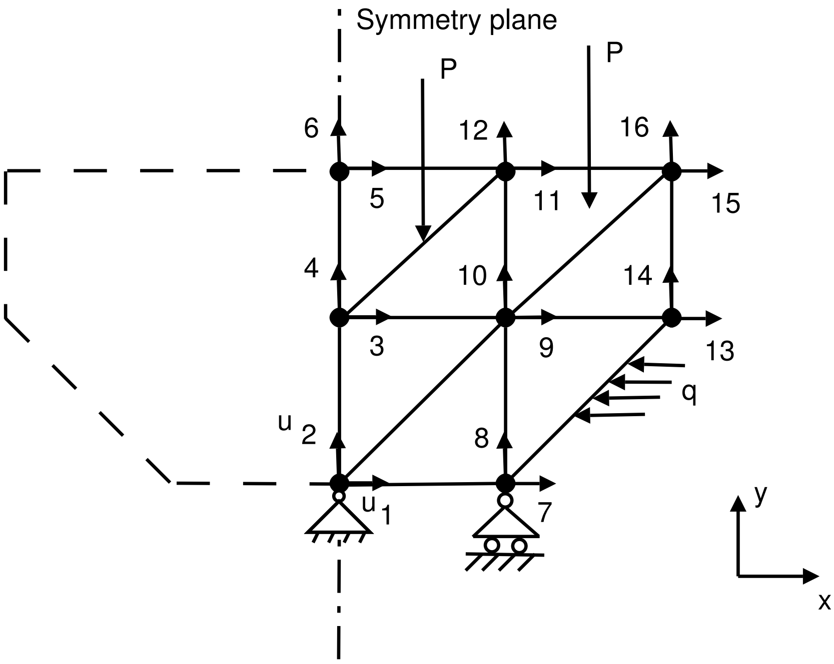
b) Derive the finite element equation.

c) A part of a domain should be modelled using four node quadratic elements. Suggest an approximation for that element. (You may assume that the element sides are parallel with the coordinate axes.)

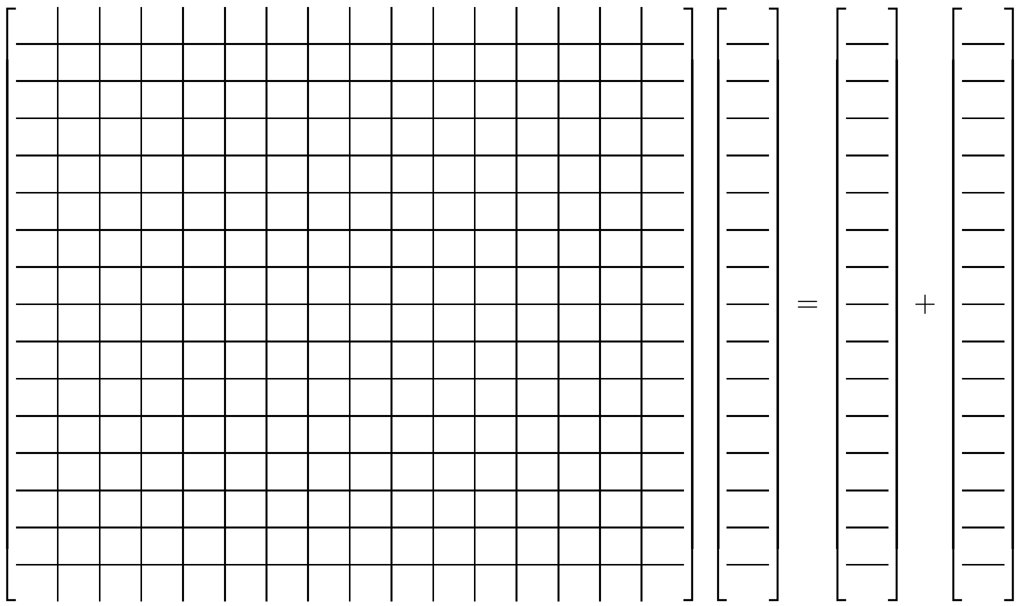
d) Does the proposed approximation fulfill the convergence criterion ?

e) Does the proposed approximation involve any parasitic terms ?

Problem 4: (12p)



A symmetric structural problem is modelled with 3-node elements and loaded with point loads P and a distributed load q. The right hand side of the structure has 16 degrees of freedom. The matrix relation found from the FE-formulation is given by $\boldsymbol{K}\boldsymbol{a} = \boldsymbol{f}_b + \boldsymbol{f}_l$, where the stiffness matrix is denoted $\boldsymbol{K}$, nodal vector $\boldsymbol{a}$, load vector $\boldsymbol{f}_l$ and the boundary vector with $\boldsymbol{f}_b$.



Mark with an x for components known and different from zero, and with 0 for components equal to zero and with ? for unknown components.

Problem 5: FHLF01 (12p)

The finite element formulation for heat conduction takes the form:

$$\left(\underbrace{\int_V \boldsymbol{B}^T\boldsymbol{D}\boldsymbol{B}dV}_{\boldsymbol{K}} + \underbrace{\int_{S_c} \alpha\boldsymbol{N}^T\boldsymbol{N}dS}_{\tilde{\boldsymbol{K}}}\right)\boldsymbol{a} = \underbrace{\int_{S_c} \boldsymbol{N}^T\alpha T_\infty dS}_{\boldsymbol{f}_c} \underbrace{- \int_{S_g} \boldsymbol{N}^T q_n dS - \int_{S_h} \boldsymbol{N}^T h dS}_{\boldsymbol{f}_b} + \underbrace{\int_V \boldsymbol{N}^T Q dV}_{\boldsymbol{f}_l}$$

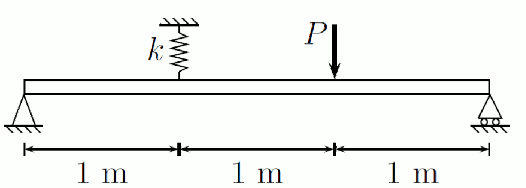
where S_c is the boundary where convection applies, S_h is the boundary where the heat flow is prescribed and S_g is the boundary where the temperature is prescribed. $\boldsymbol{D}$ is symmetric and positive definite.

a) Show that the matrix $\boldsymbol{K}$ is not invertible.

b) Assume that $\alpha = 0$, i.e. $\boldsymbol{K}\boldsymbol{a} = \boldsymbol{f}$. Show that the system $\boldsymbol{K}\boldsymbol{a} = \boldsymbol{f}$ can be solved after essential (Dirichlet) boundary conditions have been imposed. (You may assume that homogeneous boundary conditions prevails)

Problem 5: FHLF10 (12p)

A three meter long beam is subjected to a concentrated load, P as shown in the figure. At a distance of 1m from the left support a linear spring (with spring constant k) is attached. Determine the force in the spring due to the load P. The bending stiffness of the beam is EI. Use standard beam deflection formulas (sv. elementarfall) to solve the problem.



Some hints that <u>might</u> be helpful

Hint: Green-Gauss's theorem states:

$$\int_A \phi div \boldsymbol{q} dA = \oint_{\mathcal{L}} \phi \boldsymbol{q}^T \boldsymbol{n} d\mathcal{L} - \int_A (\boldsymbol{\nabla}\phi)^T \boldsymbol{q} dA$$

Hint: Isoparametric mapping: Isoparametric mapping (2D) is based on $x = \boldsymbol{N}^e(\xi,\eta)\boldsymbol{x}^e$ and $y = \boldsymbol{N}^e(\xi,\eta)\boldsymbol{y}^e$ where $\boldsymbol{N}^e$ represents the shape functions and $\boldsymbol{x}^e$ and $\boldsymbol{y}^e$ the element coordinates.

Hint: A quadratic matrix is positive semidefinite if

$$\boldsymbol{a}^T\boldsymbol{K}\boldsymbol{a} \geq 0, \quad \forall \boldsymbol{a}, \quad \text{and} \quad \boldsymbol{a}^T\boldsymbol{K}\boldsymbol{a} = 0 \quad \text{for some} \quad \boldsymbol{a} \neq \boldsymbol{0}$$

Hint: The interpolation formula of Lagrange is given by

$$l_k^{n-1} = \frac{(x-x_1).....(x-x_{k-1})(x-x_{k+1}).....(x-x_n)}{(x_k-x_1).....(x_k-x_{k-1})(x_k-x_{k+1}).....(x_k-x_n)}$$

Hint: Fourier's law is given by $\boldsymbol{q} = -\boldsymbol{D}\boldsymbol{\nabla}T$

Hint: The position, ξ_i, of the integration points and weights, H_i, for n number of integration points can be found from

n	ξ_i	H_i
1	0	2
2	$\pm 1/\sqrt{3}$	1

where integration from -1 to 1 is assumed.

Hint:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & a & b \\ 1 & c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} (ad-bc) & 0 & 0 \\ -d+b & d & -b \\ c-a & -c & a \end{bmatrix}$$