Assignment in
The Finte Element Method, 2016

Division of Solid Mechanics

The task is to develop a finite element program in order to analyze a complex geometry exposed to thermal and mechanical loads. This problem should be solved using Matlab and suitable subroutines included in CALFEM.

Main task

To minimize the power loss during operation of formula 1 cars, the tolerances between the piston and the cylinder are extremely small. To achieve this, the cylinder is manufactured with a smaller radius than the piston. The difference in radius between the piston and the cylinder is Δr. A consequence of the small cylinder radius is that the piston is not able to move at low temperatures. To resolve this issue, the engine is preheated before start. The heating is done through a water system consisting of several water channels (all with the depth d). The air surrounding the engine is assumed to have the temperature T_∞. Heat transport to the engine is due to convection with the convection coefficients α_{water} and α_c for convection from water and air, respectively. The material is assumed to be isotropic with Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α and thermal conductivity k.

During preheating the temperature in the piston is assumed to be T_∞, and the heat transport between cylinder and piston is due to convection with convection coefficient α_c.

When the engine is running, the water channels are used to cool the system. In addition the cylinder is exposed to a pressure p and heat transfer due to convection with convection coefficients α_f from a combustion temperature T_f.

To be able to perform the analysis in CALFEM, the structure is modeled using plane stress conditions. In Fig. 1 the dimensions of the cylinder, thermal and mechanical boundary conditions are shown. The heating/cooling water is indicated by gray color.

Problem formulation

Determine the temperature of the heating water so that the minimum radial displacement of the cylinder wall is $\geq \Delta r$.

Determine the transient temperature evolution when the engine is running. The initial temperature is given by the solution to the pre-heating problem. Contour plots showing the temperature evolution should be presented.

Determine the stationary temperature distribution and the corresponding stress distribution of the engine during operation. The results should be presented using contour plots based on suitable extrapolations to nodal points. The stress analysis should be presented for thermal load, mechanical load and combined thermal and mechanical load.
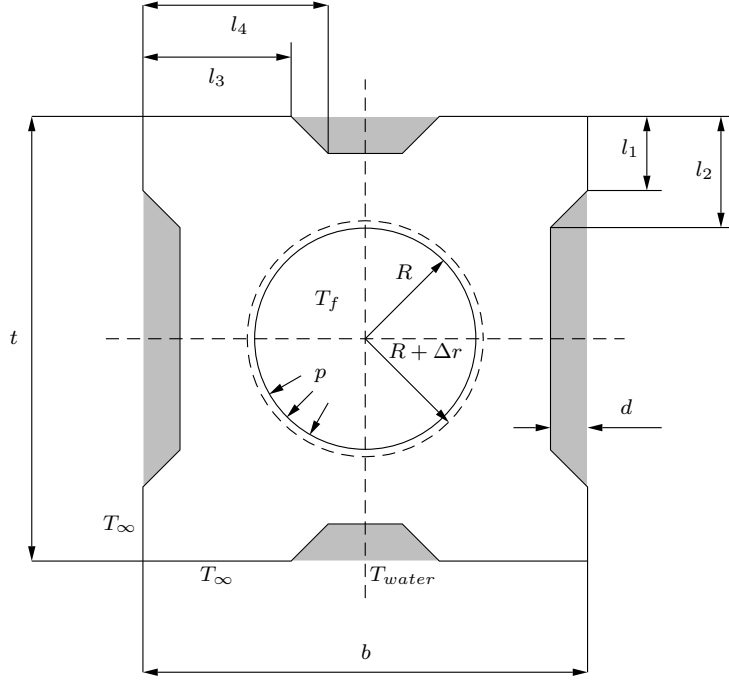
Figure 1: Dimensions of one cylinder.

Procedure

A fully implicit time integration scheme should be used. Note that the element function for forming, $\boldsymbol{C}^e$, is available at the course homepage. A suitable element is the linear triangular element. As a starting point, you have the strong formulation of the heat and the mechanical problems.

The contour plots of the stress distribution is based on the stress at the nodal points. The extrapolation of the stress in the elements to the nodal points can be done by taking the mean value of the stresses in the elements connected to a node. The following Matlab code can be used.

```
for i=1:size(coord,1)
    [c0,c1]=find(Edof(:,2:4)==i);
    Seff_nod(i,1)=sum(Seff_el(c0))/size(c0,1);
end
```

where *Seff_nod* and *Seff_el* is the von Mises effective stress at the nodal points and in the elements, respectively. *Edof* is the topology matrix associated with the temperature problem.

The geometry can be generated and meshed using the matlab toolbox *pdetool*.

Numerical data

E=72 GPa	ν=0.3	α=23·10^{-6} 1/C°
α_c=500 J/sm^2C°	α_{water}=5000 J/sm^2C°	α_f =10000 J/sm^2C°
b=187.5 mm	l_1=31.25 mm	l_2=50 mm
l_3=62.5 mm	l_4=75 mm	d=12.5 mm
R=50 mm	$\Delta r = 50 \ \mu$m	t=187.5 mm
ρ =2700 kg m^{-3}	T_f =800 C°	p=7 MPa
k =237 W/mC°	c_p =910 J/KgK	T_∞=20C°

Table 1: Numerical data

Report

A fundamental ingredient in all research is that it should be possible to regenerate the results obtained based on the report. In the present situation this implies that the appended matlab code should only be considered as supporting material. Moreover, note that one variable for grading the report is the structure of the computer code, i.e. you should choose suitable names for variables etc. A suitable structure for the report is:

- **Introduction**: Description of the problem, geometry etc. Keep this section as short as possible.

- **Procedure**: How the problems are solved (weak formulation, application of boundary conditions, thermal strains etc.). Note that you are encouraged to make references to textbooks etc. It is important to carefully present all calculations that are not available in the litterature.

- **Results**: Present the results in illustrative figures and/or tables. Note that the results should be commented such that the reader can not missunderstand the results (correct labels, units, figure texts etc.)

- **Disscusion**: A disscusion of the results. You might want to discuss sources of errors and accuracy in this section.

- **Computer Code**: Note that the code should be easy to follow and all declared variables should have intuitive names and so on.

A well structured report briefly containing all steps from the strong formulation to the FE formulation is to be returned to the Div. of Solid Mechanics no later than **May 27 at 16.00**. The reader of the report is assumed to have the same knowledge level as the author. If the report contains theoretical errors, the report is returned in order to be corrected. It is possible to obtain up to 5 points which are augmented to the points obtained at the exam in May 2016. The assignment should be approved no later than 2016-06-12. You should submit your report in PDF format to FHLF01@solid.lth.se or FHLF10@solid.lth.se. In addition to your report you should also attach your m-files in the email. Moreover, a paper version should also be handed in to the division of Solid Mechanics. Note that the bonus points obtained is only valid for the examination in May 2016.

Collaboration

The task should be solved in groups of *two* or *individually*. For further details see www.solid.lth.se and navigate to the course page.