STRUCTURAL OPTIMIZATION 2018
Dept. of Solid Mechanics

MID-TERM EXAM: 2018-02-21

A maximum of 30 points can be achieved in this examination. To pass at least 15 points are required.

Permitted aid: Pocket calculator.

Problem 1: (6p)

- Use a simple problem to explain the terms: Shape optimization, size optimization and topology optimization.
- The functional f is defined as $f(u) = \int_0^1 \sqrt{1 - \frac{1}{2}(u)'^2} dx$. Calculate the Gateaux derivative $f^{'}(u;\varphi)$.
- Explain under which conditions the adjoint method is preferable over the direct method.
- For many structural optimization problems it is useful to perform several optimizations using different initial designs. Why is this strategy useful ?

Problem 2: (6p)

a) Check if the function $f(x) = (3-x)^2 + (3-x) + (3-x)^0$ is convex.

b) Check if the function $f(\boldsymbol{x}) = (\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b})^T (\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b})$ is convex. The constant matrix $\boldsymbol{A}$ is real and indefinite.

Problem 3: (6p)

$$\mathcal{P} \begin{cases} \underset{x_1,x_2}{max} \; -(x_1-2)^2 - 2(x_2-1)^2 \\ x_1 + 4x_2 \leq 3 \\ -x_1 + x_2 \leq 0 \end{cases}$$

- Is the problem stated above convex ?
- Find a KKT point to the problem above.

Problem 4: (6p)

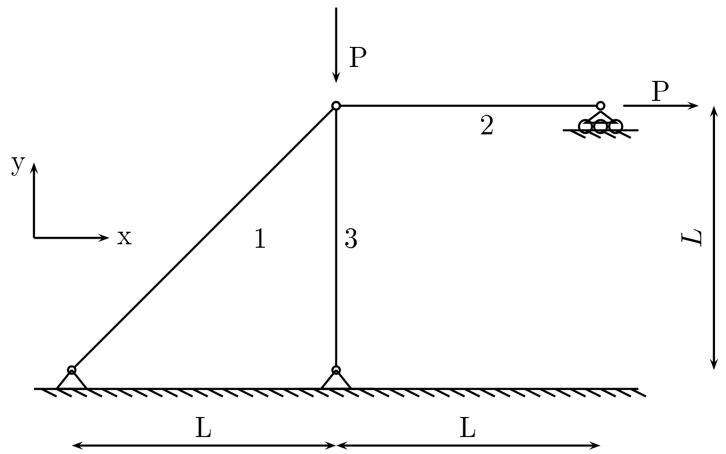
- Determine an MMA approximation to $g_0(x_1, x_2) = x_1^3 + x_2^2$ at $(x_1, x_2) = (2, -2)$.
- Choose asymptotes and move limits.
- Describe how the asymptotes should be updated when the convergence is slow.

Problem 5: (6p)

The stiffness of the structure below should be maximized by minimizing

$$min(max(|u_{1x}|, |u_{1y}|, |u_{2x}|))$$

The structure is subject to the loads $P > 0$ as indicated in the figure. The bars are made of the same material. The total volume availible for the design is V_o. Find the optimal cross-section areas of the structure. Solve the problem using Lagrangian duality.



Hints that might be useful:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & a & b \\ 1 & c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} (ad-bc) & 0 & 0 \\ -d+b & d & -b \\ c-a & -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & d & e \\ d & b & 0 \\ e & 0 & c \end{bmatrix}^{-1} = \frac{1}{abc-e^2b-d^2c}\begin{bmatrix} bc & -dc & -eb \\ -dc & ac-e^2 & de \\ -eb & de & ab-d^2 \end{bmatrix}$$

CONLIN in brief: The CONLIN approximation of g is based on a Taylor expansion in the intervening variable $y_i = 1/x_i$ if $\frac{\partial g}{\partial x_i} \leq 0$ and $y_i = x_i$ if $\frac{\partial g}{\partial x_i} > 0$

MMA in brief: The MMA approximation of g is based on a Taylor expansion in the intervening variable $y_i = \frac{1}{x_i - L_i}$ if $\frac{\partial g}{\partial x_i} < 0$ and $y_i = \frac{1}{U_i - x_i}$ if $\frac{\partial g}{\partial x_i} \geq 0$.