Assignment in
Structural Optimization, 2018

Division of Solid Mechanics

The task is to implement and analyze different optimization methods. The theory related to the methods and the results should be presented in a well structured report. The methods should be implemented in Matlab where use can be made of suitable subroutines included in the CALFEM toolbox. The developed computer code should be attached as an appendix in the report.

Problem description

An optimization of a simple geometry should be performed. Geometry and boundary conditions are illustrated in Fig. 1. The material of the beam is linear elastic, homogeneous and isotropic. Symmetry is used such that only (the right) half of the beam is analyzed. Symmetry boundary conditions is applied along the dashed line.

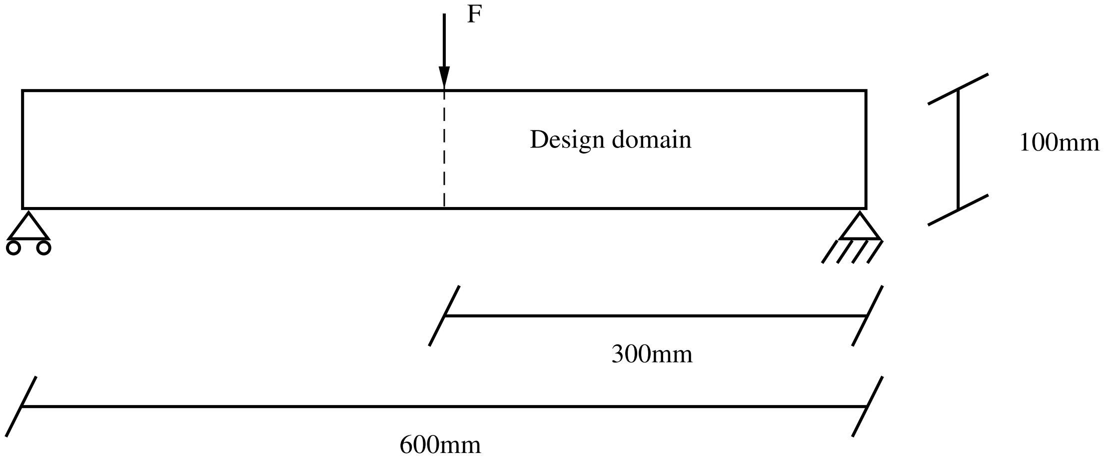

Figure 1: Illustration of geometry and boundary conditions

Problem formulation

The following topology optimization tasks should be solved for two finite element discretizations, `MBBCoarseMesh.mat` and `MBBFineMesh.mat`, both available on the course homepage. The two files contain the fixed boundary conditions `bc`, the external load `F`, coordinate matrix `coord`, degrees of freedom `dof`, nodal connectivity matrix `enod` and degrees of freedom connectivity matrix `edof`. In task a) the compliance of the beam should be minimized and in task b) the first eigenfrequency should be maximized. In the final task, c), a filtering technique should be implemented to solve the optimization problems a) and b). Choose between either MMA or the OC method for updating the design. Plane stress conditions should be used.

Compliance minimization

a) Use the SIMP algorithm to derive an optimal design. You should investivate the two discretizations and different initial values. The maximum allowed volume of the structure is $V_{max} = 0.4V_{box}$ where V_{box} is the volume defined by Fig. 1 together with an eligible thickness.

Eigenfrequency maximization

b) Establish an optimization routine to maximize the fundamental eigenfrequency for a free vibration problem. To avoid localized eigenmodes in low density regions, i.e. where the filtered design variable is small, the SIMP model needs extra attention. Therefore you should implement the interpolation forumla for the mass matrix described in [1], Section 2, according to

$$\mathbf{M}_e(\tilde{\rho}_e) = \begin{cases} \tilde{\rho}_e \mathbf{M}_e^0, & \tilde{\rho}_e > 0.1 \\ (c_1 \tilde{\rho}_e^{\,6} + c_2 \tilde{\rho}_e^{\,7}) \mathbf{M}_e^0, & \tilde{\rho}_e \leq 0.1 \end{cases} \tag{1}$$

where $\tilde{\rho}_e$ is the filtered element design variable, $\mathbf{M}_e^0$ is the constant element mass matrix, $c_1 = 6 \times 10^5$ and $c_2 = -5 \times 10^6$.

Hints that might be useful:

- The element mass matrix $\mathbf{M}_e^0$ can be calculated using the MATLAB function `planei4_m.m` available on the course homepage.
- The MATLAB function `eigenSM.m` available on the course homepage solves a generalized eigenvalue problem.
- You may neglect positive element sensitivities and replace them with zeros [1]. You may also need to stabilize the OC method by introducing move limits as shown in the Appendix.

Filtering

c) Include a filter in the SIMP algorithm. Implement *one* of the following filters

- a *density filter*,
- the Helmholz' PDE based filter.

The filtering technique should be used to solve the minimum compliance problem a) and the maximum fundamental eigenfrequency problem b). The theory describing density filters can be found in e.g. [3], and the theory describing the PDE filter can be found in [2]. Examine the influence of the length scale parameter, r. The following routines, available on the course homepage, might be useful if you choose to implement the PDE filter: `flw2i4m.m`, `plani4e_rho.m` and `getdgdrhotilde_el.m`.

Procedure

The analysis is to be performed in CALFEM. A well structured concise report of your findings should be returned to the Div. of Solid Mechanics no later than **2018-03-12 16.00**. The results

[1] replace $-\partial g_0/\partial x_e$ of (9.3) in Christensen and Klarbring by $max(0, -\partial g_0/\partial x_e)$

should be presented in the form of illustrative graphs and tables. Note that it should be possible to generate the results from the information provided in the report, i.e. numerical parameters used should be clearly stated in the report. MATLAB/CALFEM files (appendix) should be well structured and carefully commented. The reader of the report is assumed to have the same knowledge level as the author before taking the structural optimization course. It is possible to obtain up to 30 points. The task should be solved in groups of *two* (or *individually*). Keep the report as concise as possible. It is strongly recommended that you keep the report well below 10 pages excluding the appendix containing the code.

Submission

You should submit your report in **PDF** to FHLN01@solid.lth.se. In addition to your report you should also attach your m-files. Moreover, a **paper** version should be handed in to the Division of Solid Mechanics.

References

[1] Jianbin Du and Niels Olhoff. Topological design of freely vibrating continuum structures for maximum values of simple and multiple eigenfrequencies and frequency gaps. *Structural and Multidisciplinary Optimization*, 34(2):91–110, Aug 2007.

[2] B. S. Lazarov and O. Sigmund. Filters in topology optimization based on helmholz-type differential equations. *International journal for numerical methods in engineering*, 86:765–781, 2010.

[3] Ole Sigmund. Morphology-based black and white filters for topology optimization. *Structural and Multidisciplinary Optimization*, 33(4):401–424, 2007.

Appendix

To stabilize the OC method a move limit m is introduced. This is done by replacing Equation 9.5 in the course book by Christensen and Klarbring with

$$x_e(\lambda) = \begin{cases} max(\underline{\rho}, x_e - m) & \text{if} \quad \left(\frac{\alpha b_e^k}{\lambda a_e}\right)^{1/(1+\alpha)} < \quad max(\underline{\rho}, x_e - m), \\ min(\overline{\rho}, x_e + m) & \text{if} \quad \left(\frac{\alpha b_e^k}{\lambda a_e}\right)^{1/(1+\alpha)} > \quad min(\overline{\rho}, x_e + m), \\ \left(\frac{\alpha b_e^k}{\lambda a_e}\right)^{1/(1+\alpha)} & \text{otherwise.} \end{cases}$$

A suitable choice of the move limit is $m = 0.2$.