STRUCTURAL OPTIMIZATION 2019
Dept. of Solid Mechanics

MID-TERM EXAM: 2019-02-20

A maximum of 30 points can be achieved in this examination. To pass at least 15 points are required.

Permitted aid: Pocket calculator.

Problem 1: (6p)

- Express the distance between two points (x_1, y_1) and (x_2, y_2) as a functional.
- Show that the shortest distance between the two points is a straight line.

Problem 2: (6p)

a) Check if the function $f(\boldsymbol{x}) = x_1^2 - x_1x_2 - x_2^2$ is convex.

b) Check if the function $f(x) = |x|$ is convex.

c) Verify that the CONLIN approximation in $(x_1, x_2) = (1, 2)$ of $f(\boldsymbol{x}) = x_1^2 - x_1x_2 - x_2^2$ is convex.

Problem 3: (6p)

Solve the problem $\mathcal{P} \begin{cases} \underset{x_1,x_2}{min}\ x_1^2 + x_2^2 - 3x_1x_2 \\ s.t.\ \ x_1^2 + x_2^2 - 6 \leq 0 \end{cases}$

Problem 4: (6p)

The function, g_0 is given by

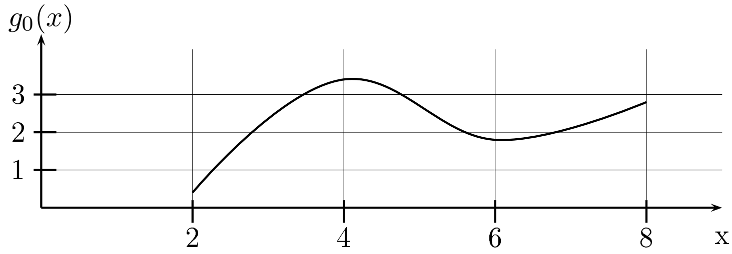


Figure 1: Illustration of the function g_0

a) Establish a MMA approximation in $x = 3$. Sketch your approximation in the attached graph. Clearly state the asymptotes that you have chosen.

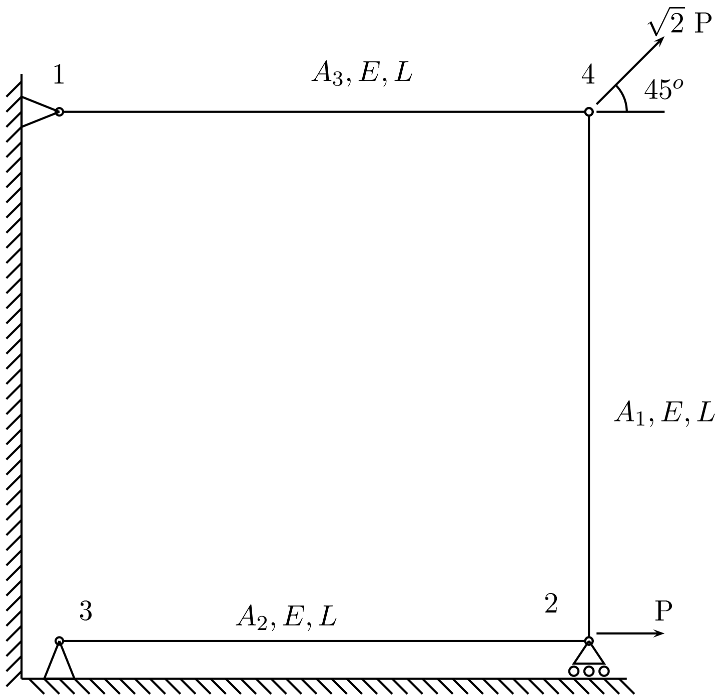
b) Establish a CONLIN approximation in $x = 5$. Sketch your approximation in the attached graph.

Problem 5: (6p)

The stiffness of the three bar truss in the figure below should be maximized. The stiffness is mesured as the 1-norm of the global displacements, i.e. we want to minimize

$$|u_{1x}| + |u_{1y}| + |u_{2x}| + |u_{2y}| + |u_{3x}| + |u_{3y}| + |u_{4x}| + |u_{4y}|$$

The total volume of the bars may not exceed V_0. The truss is subject to two loads ($P > 0$) according to the figure below. The design variables are $A_1 > 0, A_2 > 0$ and $A_3 > 0$.



a) Formulate the problem as a mathematical programming problem.

b) Is the problem convex ?

c) Solve the optimization problem using Lagrangian duality.

Hints that might be useful:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & a & b \\ 1 & c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} (ad-bc) & 0 & 0 \\ -d+b & d & -b \\ c-a & -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & d & e \\ d & b & 0 \\ e & 0 & c \end{bmatrix}^{-1} = \frac{1}{abc-e^2b-d^2c}\begin{bmatrix} bc & -dc & -eb \\ -dc & ac-e^2 & de \\ -eb & de & ab-d^2 \end{bmatrix}$$

CONLIN in brief: The CONLIN approximation of g is based on a Taylor expansion in the intervening variable $y_i = 1/x_i$ if $\frac{\partial g}{\partial x_i} \leq 0$ and $y_i = x_i$ if $\frac{\partial g}{\partial x_i} > 0$

MMA in brief: The MMA approximation of g is based on a Taylor expansion in the intervening variable $y_i = \frac{1}{x_i - L_i}$ if $\frac{\partial g}{\partial x_i} < 0$ and $y_i = \frac{1}{U_i - x_i}$ if $\frac{\partial g}{\partial x_i} \geq 0$.