STRUCTURAL OPTIMIZATION 2015
Dept. of Solid Mechanics

MID-TERM EXAM: 2015-02-24

A maximum of 30 points can be achieved in this examination. To pass at least 15 points are required.

Permitted aid: Pocket calculator.

Problem 1: (5p)

- Explain the term 'separable convex approximation'.
- Explain the difference between 'simultaneous formulation' and 'nested formulation'.
- Use a simple problem to explain the terms, shape optimization, size optimization and topology optimization.
- Make a CONLIN approximation to $g = \frac{4x}{3} + sin(x) + exp(x) - x^3$ at x=2

Problem 2: (6p)

Consider the optimization problem $\mathcal{P} \begin{cases} \underset{u}{min}\ J(u) = \underset{u}{min} \int_a^b (2u^2 + (u'))dx \\ u(a) = u_a, \\ u(b) = u_b, \\ u \in C^2[a, b] \end{cases}$

When solving $\mathcal{P}$, the functional derivative $J'(u, \varphi)$ is needed.

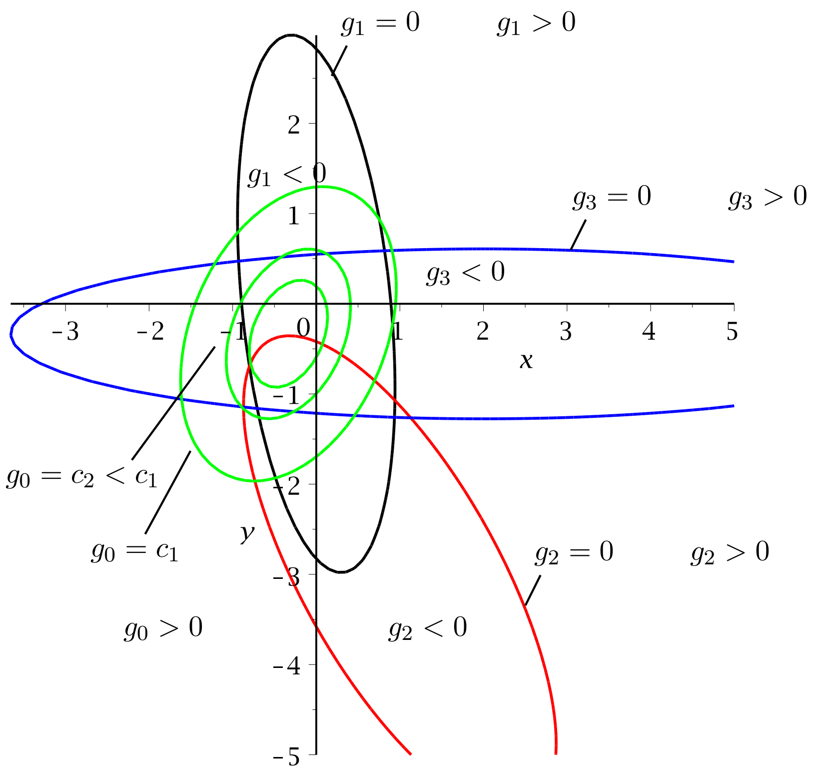
a) What is the functional derivative $J'(u, \varphi)$

b) Use the definition of the Gateux derivative to show your result in a)

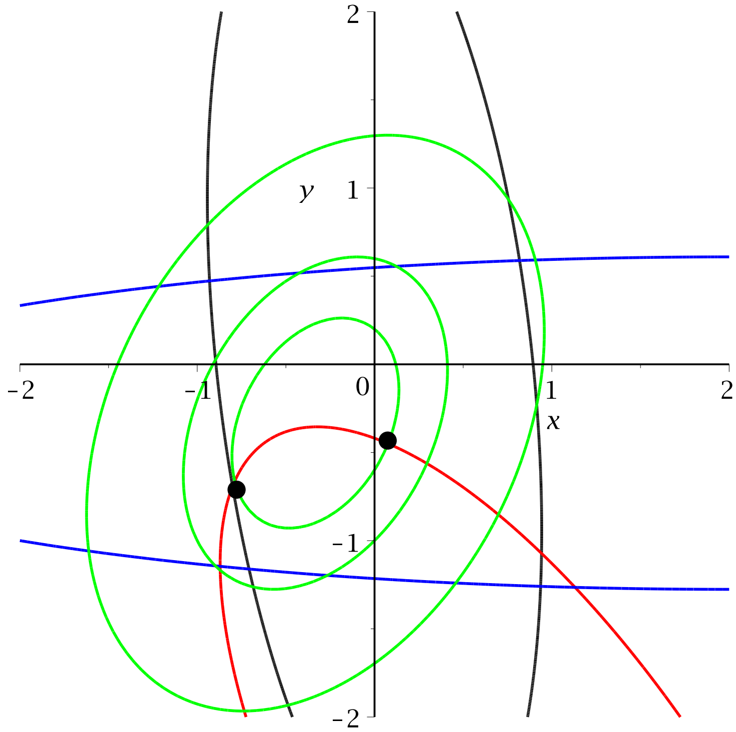
Problem 3: (7p)

Consider the following optimization problem: $\mathcal{P}\begin{cases} \min\limits_{x_1,x_2} g_0 \\ g_1 \leq 0 \\ g_2 \leq 0 \\ g_3 \leq 0 \end{cases}$

The functions g_i that defines the problem is shown below.



a) Indicate the feasible set in the figure above. Hand in this paper and do not forget to indicate your name.



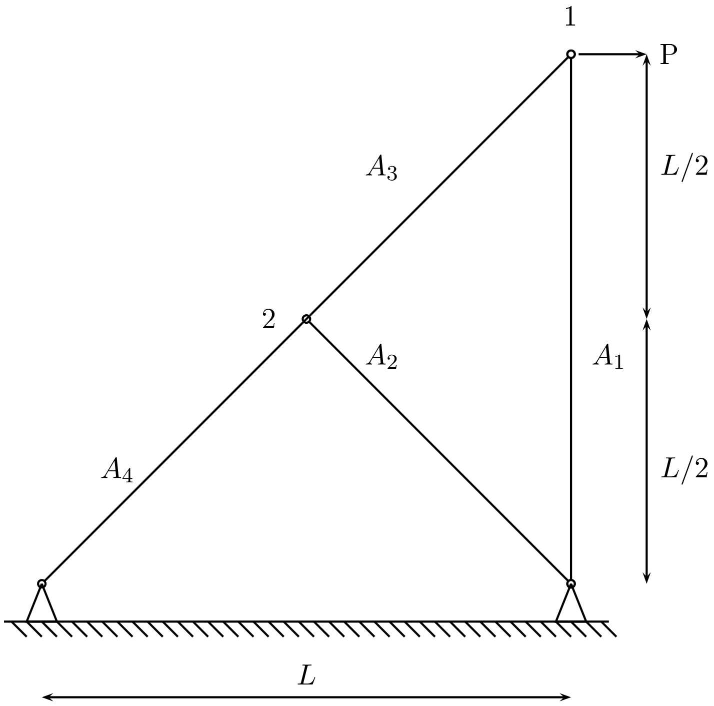
b) Use the figure above to determine if the two points (black dots) shown in the figure b) are KKT points. As usual, your answer should be well motivated. Hand in this paper and do not forget to indicate your name.

Problem 4: (7p)

The stiffness of the truss in the figure below should be maximized. The stiffness is mesured as the 1-norm of the global displacements, i.e. we want to minimize

$$|u_{1x}| + |u_{1y}| + |u_{2x}| + |u_{2y}|$$

The total volume of the bars may not exceed V_0 and the Youngs modulus is E. The truss is subject to the load ($P > 0$) according to the figure below. The design variables are A_1, A_2, A_3 and A_4.



a) Establish the matrix connecting the external forces to the truss forces.

b) The optimization problem can be stated as $\mathcal{P}\begin{cases} \min\limits_{A_1,A_2,A_3,A_4} \frac{2}{A_1} + \frac{\sqrt{2}}{A_3} + \frac{2\sqrt{2}}{A_4} \\ \sqrt{2}A_1 + A_2 + A_3 + A_4 - \bar{V} \leq 0 \end{cases}$

Solve the optimization problem using Lagrangian duality.

Problem 5: (5p)

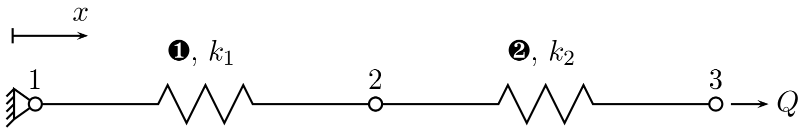


Figure 1: Illustration of a two connected springs loaded in tension.

The equilibrium for the two spring system depicted above is given by

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} Q_2 \\ Q_3 \end{bmatrix} \tag{1}$$

In a stiffness optimization the compliance is to be optimized, i.e. $C = Q \cdot u_3$.

- Use the adjoint method to calculate the sensitivity of C with respect to k_1.

Hints that might be useful:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & a & b \\ 1 & c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} (ad - bc) & 0 & 0 \\ -d + b & d & -b \\ c - a & -c & a \end{bmatrix}$$

CONLIN in brief: The CONLIN approximation of g is based on a Taylor expansion in the intervening variable $y_i = 1/x_i$ if $\frac{\partial g}{\partial x_i} \leq 0$ and $y_i = x_i$ if $\frac{\partial g}{\partial x_i} > 0$

MMA in brief: The MMA approximation of g is based on a Taylor expansion in the intervening variable $y_i = \frac{1}{x_i - L_i}$ if $\frac{\partial g}{\partial x_i} < 0$ and $y_i = \frac{1}{U_i - x_i}$ if $\frac{\partial g}{\partial x_i} \geq 0$.