STRUCTURAL OPTIMIZATION 2016
Dept. of Solid Mechanics

MID-TERM EXAM: 2016-02-24

A maximum of 30 points can be achieved in this examination. To pass at least 15 points are required.

Permitted aid: Pocket calculator.

Problem 1: (6p)

- Explain the advantages of MMA compared to CONLIN.
- The functional f is defined as $f(u) = \int_0^1 \sqrt{1-(u)'^2}dx + 6u(0) + 13$. Calculate the Gateaux derivative $f'(u;\varphi)$.
- Explain for which conditions the adjoint method is preferrable compared to the direct method. Why ? Motivate !

Problem 2: (6p)

Consider the objective function

$$g_0(x_1, x_2) = (x_1 - 1)^2 + x_2^2$$

which is subject to minimization. The constraints for the optimization problem is given by

$$\begin{aligned}
&g_1 = x_1 + x_2 - 5 \leq 0 \\
&0 \leq x_1 \leq 10 \\
&0 \leq x_2 \leq 10
\end{aligned}$$

- Make a SLP approximation in $(x_1, x_2) = (1.5, 0.5)$ and $(x_1, x_2) = (0.5, 0.5)$
- Make a CONLIN approximation in $(x_1, x_2) = (1.5, 0.5)$ and $(x_1, x_2) = (0.5, 0.5)$
- Make a MMA approximation in $(x_1, x_2) = (1.5, 0.5)$ and $(x_1, x_2) = (0.5, 0.5)$

When applicable, choose suitable numerical parameters.

Problem 3: (6p)

Consider the following optimization problem: $\mathcal{P}\begin{cases} \min\limits_{x_1,x_2} x_1^2 + 2x_2^2 \\ x_1 + x_2 \geq 3 \\ x_2 - x_1^2 \geq 2 \end{cases}$

What are the KKT conditions associated with $\mathcal{P}$? Is the problem convex ?

Problem 4: (6p)

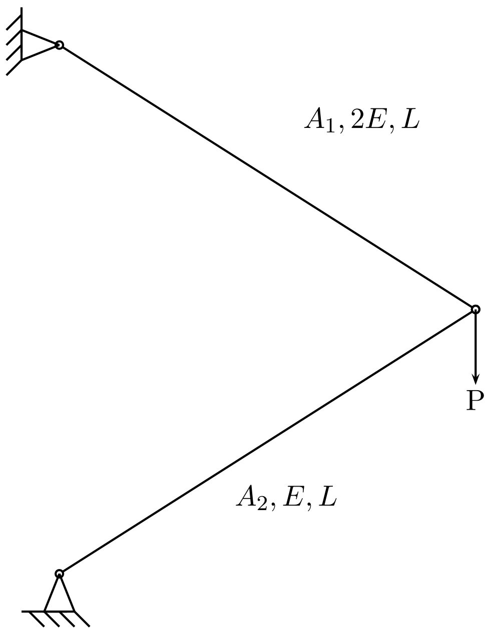
Stiffness optimization of a truss implies minimization of the compliance $C(\boldsymbol{x}) = \boldsymbol{F}^T\boldsymbol{u}(\boldsymbol{x})$ where the components, x_i, of the design vector $\boldsymbol{x}$ represents the cross section areas of the bars. It is assumed that the external load, $\boldsymbol{F}$ is independent of the design variables, x_i. The equilibrium is described by $\left(\sum_i^{nelm} \boldsymbol{K}^i\right)\boldsymbol{u} = \boldsymbol{F}$.

Show that the sensitivity $\frac{\partial C}{\partial x_i}$ can be expressed in terms of the strain energy, $w = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\epsilon}$, in element i.

Note that the expanded element matrix, $\boldsymbol{K}^i$, scales linearly with the cross section area.

Problem 5: (6p)

The two-bar truss depicted below is subject to minimization of the compliance.



Find the optimal stiffness for the constraints $|A_i| \leq V_o/2L$ and the volume $V \leq V_o$. The design variables in the optimization are the cross section areas, A_i. The vertical distance between the supports is L. Use Lagrangian duality.

Hints that might be useful:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & a & b \\ 1 & c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} (ad-bc) & 0 & 0 \\ -d+b & d & -b \\ c-a & -c & a \end{bmatrix}$$

CONLIN in brief: The CONLIN approximation of g is based on a Taylor expansion in the intervening variable $y_i = 1/x_i$ if $\frac{\partial g}{\partial x_i} \leq 0$ and $y_i = x_i$ if $\frac{\partial g}{\partial x_i} > 0$

MMA in brief: The MMA approximation of g is based on a Taylor expansion in the intervening variable $y_i = \frac{1}{x_i - L_i}$ if $\frac{\partial g}{\partial x_i} < 0$ and $y_i = \frac{1}{U_i - x_i}$ if $\frac{\partial g}{\partial x_i} \geq 0$.